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Evaluation of Probabilistic Methodology for Predicting Satellite Tracking Resources

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This thesis was prepared under the direction of the candidate's thesis committee chairman, Dr. Hamilton Hagar, Department of Electrical & Systems Engineering and co-chairman Dr. Yechiel Crispin, Department of Aerospace Engineering, and has been approved by the members of his thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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ABSTRACT

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This research evaluates a probabilistic methodology for estimating the ability of satellite tracking networks to provide tracking and data acquisition services to large constellations of satellites. This approach, developed by Hagar is evaluated using Monte Carlo simulations of optimal satellite contact scheduling on a tracking network for a certain class of satellites. The actual results of the scheduled Monte Carlo simulations were then compared to the predicted values computed with Hagar's methodology for a range constellation and network sizes. Comparison methods include percent difference, a Wilcoxon signed ranks test and a Mann-Whitney U test.

The Monte Carlo simulations were run for only low earth orbit (LEO) satellites in circular orbit at random altitudes ranging from 180km to 1000km, and inclinations from near equatorial to near polar. For each Monte Carlo sample the orbit plane orientations and initial satellite positions were randomly generated. Ninety-six different cases were simulated and compared to their respective counterparts using the probabilistic approach. The results indicate that the probabilistic method is not finished. Although the method is fair in its approximation of network capabilities it lacks the accuracy to be used as a single tool for analysis of network capabilities. With additional research and adjustment the method could give satellite network users and planners a useful tool for predicting the ability of tracking and data acquisition networks to meet current and projected satellite tracking needs.

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INTRODUCTION

Problem Statement

From the time the first satellite went into space until today there has been a need to communicate with satellites. The difference is that today there is a much larger number of satellites to communicate with. With the number of satellites to communicate with growing every year an important consideration arises. "Are there enough resources on the ground to sufficiently track and communicate with additional satellites?" This concern grows when considering the fact that satellite network operators must schedule hundreds of contacts between many unique satellites and ground stations every day. In order to determine if the existing ground stations can handle more satellites the satellites must be "test-scheduled" into the network before they ever leave the ground. This raises the problem of knowing exact launch times and dates, which determine key orbital elements, well in advance of actually launching. This is a tall order because of launch date/time uncertainties and delays (e.g., schedule slips, equipment troubles, bad weather, etc.).

To overcome this uncertainty Hagar [1] developed a basic probabilistic approach for estimating the probabilities of being able to schedule a specified set of satellites across a given network. This approach, although currently limited in its applicability, offers a potential approach for estimating whether or not a network has the likely capacity to support tracking and data acquisition (TDA). The work in this paper focuses on assessing the viability of the probabilistic approach.

Previous Research

In 1994, Hagar [2, 3] at Veridian Corporation produced a sub-optimal methodology that provided a single point solution for this tracking resource prediction problem. The method was based on a modification of the "greedy activity selector" algorithm [4]. Although the methodology was sub-optimal, it proved to be capable of accomplishing predictive allocation of tracking resources for all classes of satellites (including Earth-orbiting and deep space missions). Its major shortcoming was that it required extremely long computer run times, which became prohibitive when Monte Carlo methods were considered to accommodate the omni-present uncertainties accompanying such problems.

Burrowbridge [5] showed that a sub-class of the satellite contact-scheduling problem could be solved optimally, also based on the greedy selector algorithm. This sub-class accommodated only circular orbit satellites in the range of 300 to 1000 kilometer altitudes, but proved to be optimal in the sense of minimizing tracking resources. However, her approach was oriented to the scheduling of precisely known requirements, and did not incorporate uncertainties inherent in satellite launch dates, orbital elements, and contact requirements; nor did it accommodate uncertainties in tracking facility availability.

General Objective

The basic objective of this investigation is to determine whether a large suite of satellites can be scheduled across a satellite tracking network using a probabilistic algorithm. Is it possible to obtain an indication of the probability that a specific suite of satellites can be scheduled across a network, and if so, how good is such an approach?

The basic satellite scheduling problem concerns

1. *Visibility determination* - determining when each satellite passes within tracking range of a tracking station. Each satellite in orbit will pass within view of one or more satellite tracking stations which are located at different latitudes and longitudes across the globe. This occurs on the order of a few, to many times per day, depending upon the characteristics of the satellite orbit.[1] When a satellite rises above the station horizon it becomes a candidate for a contact with that station. Determining the particular times that a satellite rises over the horizon is not easily (if at all) solvable in closed form, and must be done practically by detailed simulation of the satellite motion – a generally time consuming process when hundreds of satellites are involved.
2. *Contact requirement* – determining whether or not each satellite requires a contact with that tracking station. The requirements for contact by each satellite with a ground station can vary widely, ranging from a very few, to many times per day.

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[1] Generally, the lower the satellite altitude the more frequently the satellite passes in range of the station. However, this varies significantly: a station located near the poles will experience many passes from a polar orbiting satellite – as many as one per orbit – but fail to see satellites in low inclination orbits. Yet, a geostationary satellite will remain essentially stationary over the same Earth subpoint, thereby always being in view of stations within its nearly global hemispheric coverage.

3. *Contention/conflict* – determining which satellites within range of the tracking station are in contention depends upon both the time they appear in view of the station (per 1 above) as well as whether or not a contact is required (per 2).
4. *Scheduling priority* – determining which of any contending satellites gets the contact. A particular scheduling algorithm must be applied, of which there may be many. For the purposes here a so-called first come, first served algorithm is used (for reasons that are explained later).

Each of these functions must be treated probabilistically, and the subsequent sections describe the essential algorithms for accomplishing this.

Once such a probabilistic methodology is established, it is tested using

1. A Monte Carlo simulation strategy that randomly selects and generates many satellites over a particular range of orbit characteristics, and
2. The application of statistical tests to ascertain the quality with which the probabilistic approach may accurately represent the likelihood of solving an actual scheduling problem.

BACKGROUND THEORY

Probabilistic Approach

The following is a basic look at Hagar's probabilistic method and the ideas behind it. The overall basics are included here but for an in depth description see Appendix A. The essence of Hagar's method is to compute the probability of (1) a satellite being within communications range of a candidate tracking station, and subsequently (2) the probability associated with the satellite requiring a contact, and (3) probabilities of resolving any schedule conflicts with other satellites competing for tracking time at the same station.

Hagar's method entails some basic (and limiting) assumptions:

1. Satellites are restricted to circular low Earth orbits.
2. Satellite dynamics are modeled as two body, Keplerian orbits.
3. Orbit altitudes range from 180 km to 1000 km.
4. Satellite contact requirements are specified in terms of contacts required per day (typically 6, and ranging to a more demanding 10). Only one contact can be achieved in any single satellite pass over a tracking station.

Probability of a Satellite in Communications Range

The region surrounding a tracking station is modeled as a circle formed by the intersection of an Earth-centered cone with the Earth's surface. This cone is formed as a result of the intersection of the satellite orbit with the station's local horizon, as shown in figure 1.

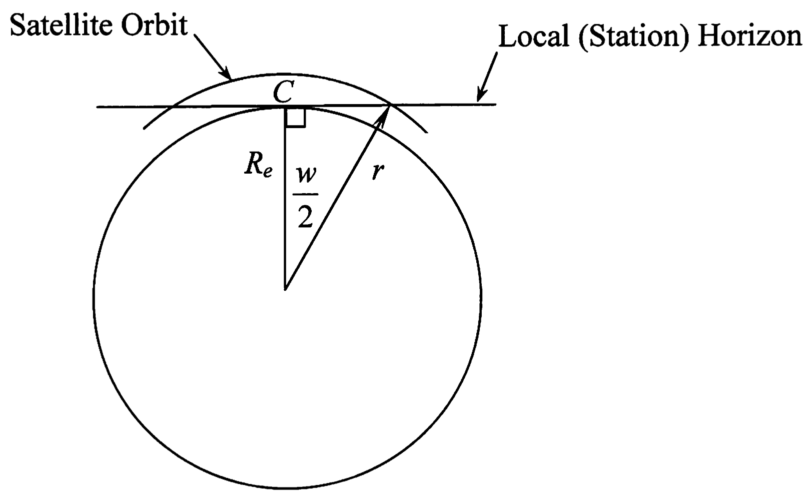

Figure 1 - Station Field of View

C represents the station location at some point on the Earth's surface. The angle $\frac{w}{2}$ is the half angle of the cone. Whenever the satellite orbit ground track passes through this circular region the satellite is considered to be within the station field-of-view (FOV). This is further illustrated in figure 2 which shows hypothetical orbit ground tracks of a single satellite just skirting the eastern and western edges of the FOV.

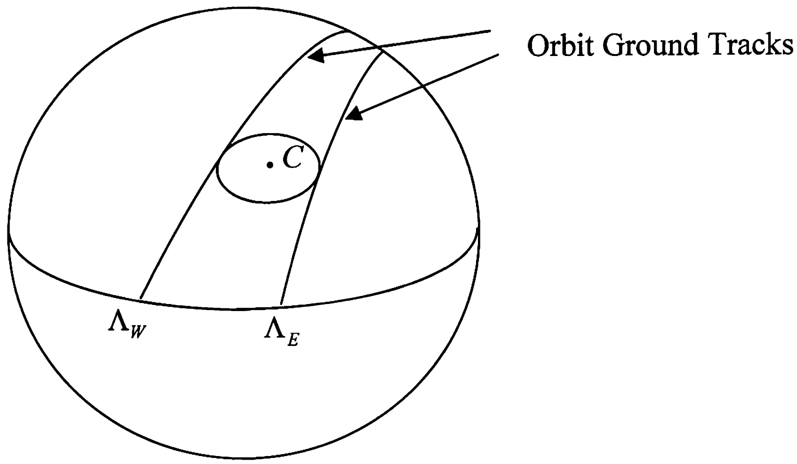

Figure 2 - Station FOV and Orbit Passes

If the Earth were stationary, these ground tracks would coincide with the orbit plane (assuming two-body orbital mechanics). While this is obviously not the case, for low Earth orbits, the longitude error in ground track location amounts to around 3 degrees for a mid-latitude station.

The longitude of the ascending nodes crossings, Λ, (points of intersection of the orbit plane with the equator) establish bounds for the orbit planes such that whenever the plane lies within this bounded region it also intersects the station FOV region. These ascending nodes crossings, Λ, must be calculated using spherical trigonometry.

In order to calculate Λ, Hagar used previous work by Wilkinson [6]. A description of this particular work is included here but for the original work consult the referenced journal [6].

The first step to finding the ascending node crossings is to find into which of the following five groups the satellite/station combination involved falls. The five groups are based on the latitude of the station $[\phi_C]$, the orbit inclination of the satellite $[\iota]$ and the half angle of the FOV cone $[\frac{w}{2}]$.

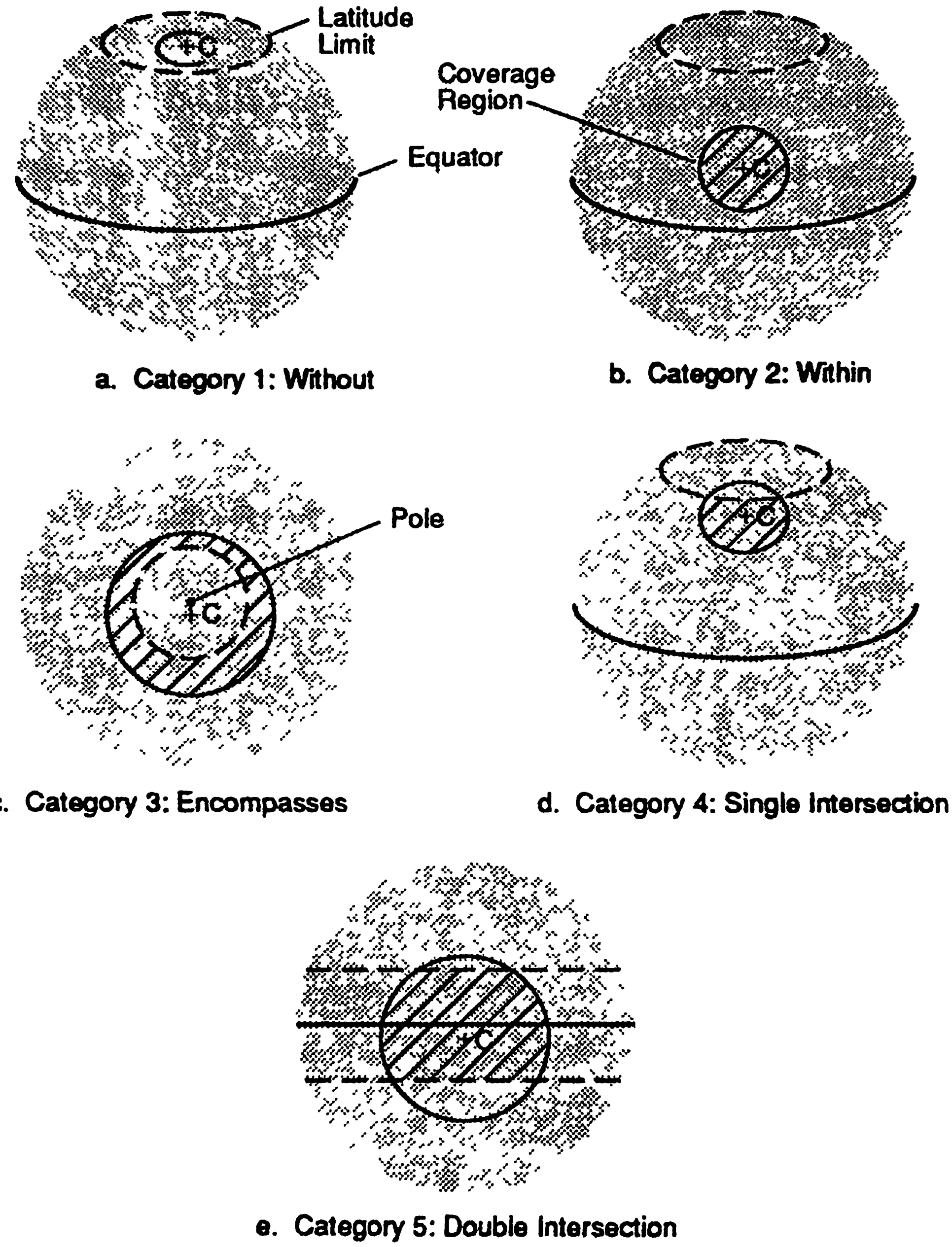

Figure 3 - Coverage Categories [6]

1. The coverage region is completely outside the latitude limits: $\sin(|\phi_C| - \frac{w}{2}) \geq \sin i$.

2. The region is completely within the latitude limits:

   $\sin(|\phi_C| + \frac{w}{2}) \leq \sin i$ *and* $\cos(|\phi_C| + \frac{w}{2}) \geq 0$.

3. The region fully encompasses either the northern or southern latitude limit:

   $\sin(|\phi_C| + \frac{w}{2}) \leq \sin i$ *and* $\cos(|\phi_C| + \frac{w}{2}) \leq 0$.

4. The region intersects either the northern or southern latitude limit, but not both:

   $\sin(|\phi_C| + \frac{w}{2}) > \sin i$ *and* $-\sin i \leq \sin(|\phi_C| - \frac{w}{2}) < \sin i$.

5. The region intersects both the northern and southern latitude limits:

   $\sin(|\phi_C| - \frac{w}{2}) < -\sin i$.

Obviously categories 1, 3 and 5 are straightforward cases where the satellite will always (Categories 3 and 5) come into contact with the station or never (Category 1) come into contact. For these cases there is no need to calculate the ascending node crossings because the probability of crossing the station is either 1.0 or 0.0.

For categories 2 and 4 the tangents are calculated as follows, using figure 4 below for reference. The point E is at the instantaneous longitude of the ascending node at the time the satellite is at the tangent point. The general procedure is illustrated here by showing the steps for the northerly ascending tangent: first, triangle EFN yields the identity $\sin\gamma = \cos i / \cos\phi_N$; apply the law of cosines to triangle PNC and replace

$\sin\gamma$ with the previous to obtain an equation for ϕ_N; compute $\Delta\lambda_N$ using the law of cosines on the same triangle; finally, compute λ_N from the right spherical triangle EFN.

The equations that follow are correct for all cases, ascending and descending passes. Note the subscript z is replaced with N for tangents nearest the equator and S for tangents nearest the pole, also the "sign" function is defined as $sign(x) = +1$ for $x \geq 0$, and $= -1$ for $x < 0$. The subscripts A and D refer to ascending and descending passes respectively.

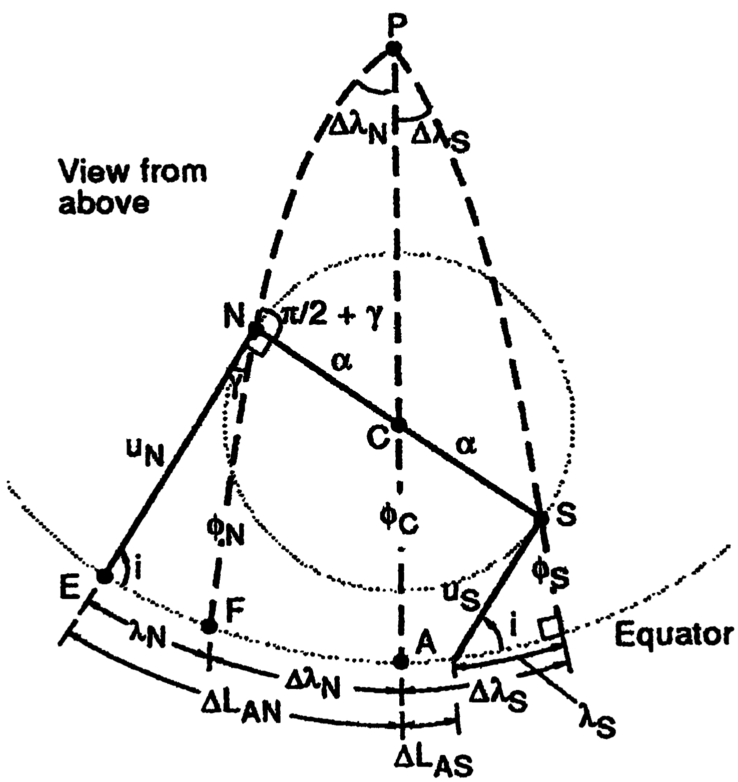

Figure 4 - Bounding Ascending Nodes [6]

$$j_N = +1$$
$$j_S = -1$$
$$q = sign(\cos i \sin\phi_C)$$

$$\sin\phi_z = (|\sin\phi_c| + j_z|\sin\phi_c|\sin\frac{w}{2}) / \cos\frac{w}{2}$$

$$\Delta\lambda_z = \cos^{-1}\left[\frac{\cos\frac{w}{2} - |\sin\phi_c|\sin\phi_z}{\cos\phi_c\cos\phi_z}\right]$$

$$\lambda_z = \sin^{-1}\left[\frac{|\cos i|\sin\phi_z}{\sin i\cos\phi_z}\right] = \sin^{-1}\left[\frac{\tan\phi_z}{\tan i}\right]$$

$$\Delta L_{Az} = -q(\lambda_z + j_z\Delta\lambda_z)$$

$$\Delta L_{Dz} = -q[\pi - (\lambda_z + j_z\Delta\lambda_z)]$$

$$L_{Az} = \lambda_C + \Delta L_{Az}$$

$$L_{Dz} = \lambda_C + \Delta L_{Dz}$$

For ascending passes:

$$L_{AN} = \Lambda_W$$

$$L_{AS} = \Lambda_E$$

For descending passes:

$$L_{DN} = \Lambda_E$$

$$L_{DS} = \Lambda_W$$

Now that the value of Λ can be found for all cases a simple equation can be applied to determine the probability of a station passes. If it is assumed that the right ascension of the ascending node of any given orbit is random and uniformly distributed over $[0, 2\pi]$, then the probability that satellite i will pass through station j's FOV region can be modeled as

$$p_{\Lambda ij} = 2\left(\frac{\Lambda_E - \Lambda_W}{2\pi}\right) = \frac{\Lambda_E - \Lambda_W}{\pi}$$

Because passages can occur on both ascending as well as descending orbit passes, there are essentially two daily opportunities to pass through a station region.

Given that a satellite orbit plane passes through the station FOV region, it is necessary to establish a probability that the satellite itself lies within the FOV – i.e., the probability that the station can actually "see" the satellite. Again the assumption is invoked that the location of the satellite is modeled as a uniformly distributed random variable over a single orbit revolution. This can be modeled as the time spent within the FOV as a fraction of one orbital period (or equivalently, for circular orbits, the fraction of 2π the satellite spends within the FOV). However, depending upon where the satellite crosses into the region, this time (or angle) can vary[2]. A solution for this is to use the expected value of w, found in Appendix A to be

$$\hat{w} = w\frac{\pi}{4}$$

In terms of the pass time, this corresponds to

$$\hat{\tau} = \tau\frac{\pi}{4}, \text{ where } \tau = \frac{w}{\dot{\theta}} = \frac{w}{\sqrt{\frac{\mu}{r^3}}}.$$

The denominator in the latter expression is the Keplerian mean motion for a satellite of circular orbit radius, r (μ is the Earth's gravitational parameter).

Using either of these expressions yields the probability of satellite i being within station j's FOV, given that its orbit plane intersects the FOV:

$$p_{\theta ij} = \frac{\hat{w}}{2\pi} = \frac{\hat{\tau}}{2\pi}\sqrt{\frac{\mu}{r^3}}$$

The above two simple probability expressions can be multiplied[3] to give the instantaneous probability of satellite i being within station j's FOV:

$$p_{Oij} = p_{\Lambda ij} p_{\theta ij}$$

[2] Note that there is a minimum tracking time (360 seconds for this research) associated with all satellites. This minimum time corresponds to the amount of time a satellite needs in a FOV to have viable contact with a ground station. An effective FOV is determined using this minimum time to make sure enough time is available for a contact. A visual depiction of the effective FOV can be found in figure 27, page 84.

(The subscript O is intended to mean an "opportunity" for contact.)

Probability Satellite Contact Required

As described in the assumptions (see **BACKGROUND THEORY, Probabilistic Approach**), satellite contacts are given typically as required contacts per day, n_{R_i}. The number of opportunities to carry out a contact depends upon the satellite's being within the station FOV. This, in large part, depends upon satellite altitude: the higher the satellite, the fewer the number of opportunities, but the longer the time for any single opportunity[4]. In many instances the number of opportunities (or "visibilities") in a day may exceed the required number of contacts. Absent any definitive information, the scheduling of a contact during any given station pass is assumed to be (again) a uniformly distributed random variable. Thus in instances where the total number of expected opportunities per day N_{O_i} (for satellite *i*) exceeds the number of required contacts n_{R_i} per day (for satellite *i*), the probability of requesting scheduling of a contact for satellite i, given that it is over a station is:

$$p_{ci} = \frac{n_{Ri}}{N_{O_i}}$$

[3] The location of the orbit plane and the satellite's location within the orbit are modeled as independent events, even though they clearly are not.

[4] Obviously, in the case of geostationary satellites there is one opportunity of infinite duration – the satellite is always in view. Higher than this altitude the process is reversed until, at an infinite distance the number of opportunities approaches one per day (the satellite becomes "stationary", and the station does all the moving).

$N_{Oi} = \sum_{j=1}^{m} n_{Oij}$; $n_{Oij} =$ number of opportunities of satellite i with respect to station j.

Further, $n_{Oij} = p_{Oij} N_i$; N_i = # of orbit revs per day of satellite i. (See the section,

Calculation of the tracking opportunity probabilities.)

Then the corresponding instantaneous probability of requesting contact scheduling for satellite i becomes[5]:

$$p_{ij} = p_{Oij} p_{ci} = p_{\Lambda ij} p_{\theta ij} p_{ci}$$

[5] Again, the events are assumed independent.

Probability of Conflict

Because the general problem deals with many satellites sharing limited resources (tracking stations), at any given time there are very likely to be a number of satellites contending for tracking and data communications with the same station. This is the issue of conflict or contention for tracking resources.

Assuming that satellite *i* is within range of station *j*, the probability of satellite *k* being in contention is simply the probability $p_{kj} = p_{Okj} p_{ck} = p_{\Lambda kj} p_{\theta kj} p_{ck}$ from above. For all other satellites the probability of at least one other satellite being in conflict follows from elementary probability as

$$p_{ij}^{\times} = 1 - \prod_{k=1}^{n}{}' \bar{p}_{kj} = 1 - \prod_{k=1}^{n}{}' \left(1 - p_{kj}\right)$$

where the prime $(\ )'$ means the product is taken over all *k* except $k = i$. Here the shortened notation $p_{kj} \equiv p_{ijk}^{\times} = p_{\Omega kj} p_{\theta kj} p_{ck}$ has been used.

Similarly, the probability of *no conflict* with any other satellite at station *j* is

$$p_{ij}^{\bar{\times}} = \prod_{k=1}^{n}{}' \bar{p}_{kj} = \prod_{k=1}^{n}{}' \left(1 - p_{kj}\right).$$

In this latter case where no satellites are in contention, the probability of satellite being scheduled can be assumed equal to 1. The former case – contention – is more complicated: which satellite gets the contact?

Hagar's method does not provide a theoretical expression for the case of contention. However, this analysis has developed a reasonably accurate, partially empirical expression. The result is an expression for the total probability of scheduling satellite *i* at station j consisting of the probabilty of no contention plus the probability of conflict times an empirical factor:

$$p_{sij} \equiv p_{ij}^{\bar{\times}} + p_{ij}^{\times}\left(\frac{h_i - h_{\min}}{h_{\max} - h_{\min}}\right)^m$$

Here,

$h_i =$ orbit altitude of satellite i

$h_{\min} =$ minimum altitude of all satellites

$h_{\max} =$ minimum altitude of all satellites

$m =$ number of tracking stations in the network

Probability of Contact Scheduling

Hagar showed that the probablity of scheduling contacts for satellite i across an entire tracking network can be determined by application of a generalization of the binomial distribution (see Appendix A). This expression is[6]

$$\underline{p}_{si}\left(n_{Ri}, n_{Oij}, m, p_{sij}\right) = \sum_{x_1=0}^{n_{ri}} \sum_{x_2=0}^{n_{Ri}-x_1} \sum_{x_3=0}^{n_{Ri}-x_1-x_2} \cdots \sum_{x_{m-1}=0}^{n_{Ri}-x_1-x_2-\ldots-x_{m-2}} \prod_{j=1}^{m} \binom{n_{Oij}}{x_j} p_{sij}^{x_j} q_{sij}^{n_{Oij}-x_j} \; ; \; q_{sij} \square 1 - p_{sij}$$

This equation is subject to the important constraints:

$$\sum_{j=1}^{m} x_j = n_{Ri} \;, x_j \le n_{Oij}, \text{ and } \binom{n}{k} \equiv 0 \text{ for } k > n$$

where

$n_{Ri} =$ number of required contacts per day for satellite i

$n_{Oij} =$ number of opportunities (visibilities) for satellite i at station j

$\binom{n}{k} =$ the binomial coefficient, and $\equiv 0$ for $k > n$

The above equation accommodates the possibility that there are a number of ways in which a given satellite can have its contacts scheduled across a network. For example, consider a two-station network and a single satellite having two contact opportunities per day at each station. If the satellite requires two contacts per day, it can have both contacts scheduled at one station, both contacts scheduled at the other station, or one contact scheduled at each station. The fact that the number of contact opportunities and required contacts can vary among satellites results in the somewhat complicated expression above.

Calculation of the tracking opportunity probabilities

Using the probabilities mentioned earlier it is possible to gain an estimate of the expected number of station passes per day. During a 24 hour period each satellite makes a total of $N_i = \frac{\dot{\theta}_i}{\omega_e}$ orbit revolutions per day, where $\dot{\theta}_i$ is the rotational velocity of the satellite and ω_e is the rotational velocity of the earth. Since inertially fixed orbit planes are assumed (the Keplerian motion), the expected number of station *j* passes per day for satellite *i* is simply:

$$n_{Oij} = \frac{\dot{\theta}_i}{\omega_e} p_{Oij}$$

Once the station passes per day is known the other probabilities can be applied as necessary in order to find the expected number of scheduled activities per day.

[6] Hagar has been unable to determine whether or not this type of generalized distribution has been developed elsewhere, and refers to it as a heterogeneous binomial distribution. Note that if the individual scheduling probabilities p_{sij} are all the same, the expression reduces to the standard binomial distribution.

Method of Assessment

The following are the general steps taken in the assessment of the probabilistic method. There is a more in depth discussion of each of the steps which follows in the subsequent sections.

First the orbits of a wide variety of satellites around the Earth and their visibilities with respect to each tracking station were simulated. Initial conditions were generated based on selecting each satellite orbit's right ascension of the ascending node (Ω) and argument of latitude (u) from uniform distributions over the interval $[0,2\pi]$. The resulting visibilities represent the time periods during which communications between the satellite and tracking station can be conducted.

Next the Greedy Activity Selector (which is optimal for certain low altitude, circular satellite orbits) is applied in order to determine which satellites get scheduled during visibility periods. It will be shown later that this gives the optimal schedule for the particular situation.

This sequence is then performed for a large number of randomly generated initial conditions for each satellite. For each random sequence, the scheduling results were recorded in terms of selected performance parameters or metrics (also described below).

Finally the results for this method were compared to those obtained using the probabilistic algorithm using percent differences, a Wilcoxon signed ranks test, and the Mann-Whitney U-Test.

Orbital Mechanics

The following is a description of the orbital mechanics involved in the Monte Carlo simulations. For a look at the equations involved, refer to Appendix C, which contains a more complete description.

For a particular satellite orbit, the satellite may come within (communications) view of a tracking station at some particular time. This time depends upon the initial state (position and velocity) of the satellite. Altering the initial state of each satellite varies the corresponding rise and set times at a tracking station, and in turn its visibility times (contact durations). As satellite initial conditions are varied, a completely different scheduling problem for the network of tracking stations occurs. By repeatedly varying these scheduling "problems" the overall network of tracking stations can be assessed in terms of its probable performance. This can then be compared on a formal statistical basis, with the probability results generated by the probabilistic algorithm being assessed.

To vary the initial conditions simply vary the right ascension of the ascending node as well as the argument of latitude. Since the orbits are all assumed circular (for this assessment), these two are adequate to achieve variations in the satellite initial states.

Each satellite, depending on its initial conditions, will rise over a station at different times depending upon both the initial satellite orbit state as well as the particular orbit revolution number. Because of this the duration available for communication will also vary. Additionally, there is a minimum communications time that is required in order to have a usable contact. This minimum time is usually in the range of 5 to 7 minutes.

Scheduling Contacts

The Satellite Scheduling Problem

A difficulty in allocating tracking resources to different satellites appears due to the fact that a single tracking site may actually be in view of more than one satellite at a given time. The problem then becomes which of the simultaneously viewable satellites should get the track? The reason this is a problem is due to the fact that a scheduling choice can influence many changes later in the scheduled period. The figure below shows the potential contacts and visibility periods for three unique satellites by a single ground station.

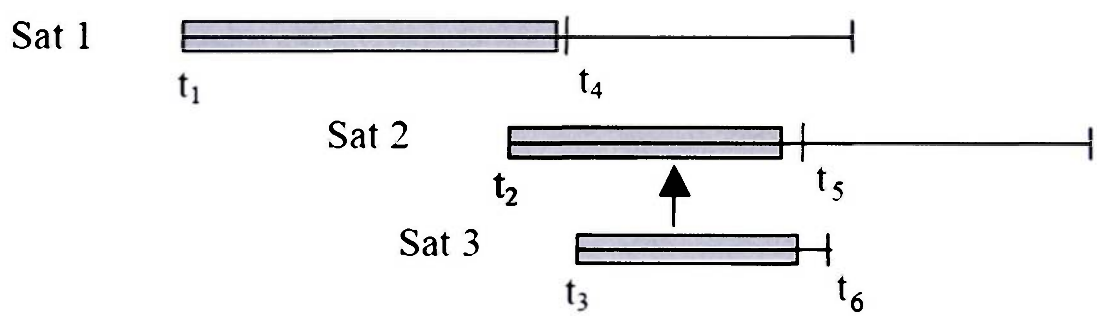

Figure 5 - Satellite Scheduling Issue

The regions shaded in figure 5 show the durations of the satellite contacts with the single lines representing the amount of time the particular satellite is visible by the station. If these 3 satellites are scheduled based on which satellite is visible first the result is a schedule which only schedules the first satellite, since all other satellites would conflict with this satellite's time. However, in the case of low altitude orbits in which each pass has only a single opportunity for scheduling, the visibility for satellite 1 is shortened to end at t_4 and the visibility for satellite 2 is shortened to end at t_5. For the resulting configuration, the optimal schedule would become satellite 1 followed by satellite 3, since satellite 2 overlaps with satellite 1. The problem is now finding an algorithm which makes the proper scheduling choice in order to optimize the schedule for all satellites. The Greedy Activity-Selector is the algorithm which was chosen to solve this problem.

Greedy Activity Selector

The Greedy Activity-Selector as defined by Cormen, Leiserson and Rivest [4] is an optimal algorithm which solves the scheduling problem of a single resource with multiple competing activities. A greedy algorithm "makes a locally optimal choice in hope that this choice will lead to a globally optimal solution". The optimality of the solution lies purely in the fact that it schedules the maximum amount of activities in a given time. The proof by Cormen, Leiserson and Rivest (repeated in Burrowbridge [5]) can be found in Appendix D. The algorithm optimizes the schedule by always choosing to schedule the activity which will leave the most remaining time to schedule other activities. This means the greedy algorithm will first order all activities by ending time. Next the algorithm will start the schedule set by choosing the first activity from the list ordered by end times. Finally, the algorithm will continue by choosing the activity which will have the earliest ending time without conflicting with any of the previous choices. Repeating the final step ensures repeatedly making a locally optimal choice. These repeated "locally optimal choices" give the desired "globally optimal solution".

Metrics

The performance parameters or metrics were chosen as a way of recording data from each method to compare. Every group of data from the probabilistic method has a counterpart from the Monte Carlo calculations giving sets of comparable data. Each group from the Monte Carlo calculations was accumulated over all iterations and averaged. Five metrics were used in the evaluation. Two are a direct indication of performance:

- Values of the total number of satellite contact opportunities
- Values of the total number of scheduled satellite contacts

Three others reflect the overall tracking network and scheduling performance:

- Feasibility ratio
- Utilization ratio
- Performance ratio

The first set is the total number of opportunities (n_O) or visibilities for the entire group of satellites used. This number simply indicates the sums of all visibilities for all satellites over all stations.

The second set recorded is the number of scheduled contacts (n_C) for all the satellites in the group being tested. This group gives the sums of all scheduled contacts between all satellites and all stations. This number will never be larger than the number of required contacts for a given situation but ideally would equal that number if all the required contacts were scheduled.

The final three sets involve ratios: the feasibility ratio, the utilization ratio and the performance ratio. The feasibility ratio is the ratio of opportunities to required contacts. A feasibility ratio higher than one indicates that there are more visibilities than required contacts. A feasibility ratio less than one indicates that an insufficient number of opportunities to schedule all the requested contacts. The higher this ratio is the better the chance will be to schedule all of the required contacts.

Feasibility ratio, $\rho_f = \frac{n_O}{n_R}$, *(# of opportunities)/(# of required contacts)*

The utilization ratio is the ratio of scheduled contacts to opportunities. This ratio indicates what portion of the visibilities is being used. This ratio will never be above 1.0.

Utilization ratio: $\rho_u = \frac{n_C}{n_O}$, *(# of scheduled contacts)/(# of opportunities)*

Finally the performance ratio is the ratio of scheduled contacts to required contacts (n_R). A performance ratio of 1.0 would indicate that all required contacts are being scheduled. This ratio will never go above 1.0 since the program stops scheduling satellites when there requirements are fulfilled.

Performance ratio: $\rho_p = \frac{n_C}{n_R}$, *(# of scheduled contacts)/(# of required contacts)*

Note that these ratios are not all independent, since the performance ratio can be formed as the product of the other two:

$$\rho_p = \frac{n_C}{n_R} = \frac{n_C}{n_O}\frac{n_O}{n_R} = \rho_u \rho_f$$

Once all the metrics were compiled comparison between the two methods was needed. Three comparison methods were used: percent differences, Wilcoxon signed ranks test and the Mann-Whitney U test.

Wilcoxon Signed Ranks Test

The Wilcoxon signed-ranks test [7] applies to two-sample designs involving repeated measures, matched pairs, or "before" and "after" measures. In this case it is the matched pairs that are interesting. Beginning with a set of paired values of X_a and X_b, which in this experiment implies a set of values from the Probabilistic method and the matching data from the Monte Carlo simulation, the following steps are completed:

- take the absolute difference $|X_a-X_b|$ for each pair;
- omit from consideration those cases where $|X_a-X_b|=0$;
- rank the remaining absolute differences, from smallest to largest, employing tied ranks where appropriate (assign the average rank for tied ranks);
- assign to each such rank a "+" sign when $X_a-X_b>0$ and a "-" sign when $X_a-X_b<0$;
- finally calculate the value of W for the Wilcoxon test, which is equal to the sum of the signed ranks.

$$W^+ = \sum(\text{Ranks associated with positive differences})$$
$$W^- = \sum(\text{Ranks associated with negative differences})$$
$$W = \min(W^+, W^-)$$

The number of signed ranks, here designated as $\mathbf{n_{s/r}}$, is equal to the number of X_aX_b pairs minus the number of pairs for which $|X_a-X_b|=0$.

When $\mathbf{n_{s/r}}$ is equal to or greater than 10, the sampling distribution of W is a reasonably close approximation of the normal distribution. In this case, the value of $\mathbf{n_{s/r}}$ is equal to 24 for all sets of data meaning the appropriate z-ratio will be calculated for all sets. The z-ratio will be described in its own section which follows.

Mann-Whitney U Test

The Mann-Whitney U test [7] is a test which is used to test whether two population distributions are identical. In this situation it will be used to determine whether or not the distributions of the Monte Carlo calculations and the probabilistic approach are identical or at least similar. The test statistic, U, is based on the ranks of observations rather than on the numerical values. When at least one of the samples contains more than 20 values a normal-approximation procedure must be used. The samples used will contain 24 values each and thus fall into this specific version of the U test.

In the U test the two samples are combined and treated as one sample. Once this is accomplished all values are ranked from 1 to 48 with the smallest value receiving a rank of 1. If two values are identical they receive the average of the combined ranks. After the values are ranked they are split back up into their respective groups where the ranks are summed. These summed ranks ΣR_1 and ΣR_2 are then used in the following equations to determine the U value. n_1 and n_2 are the number of values in each individual data set, 24 as applied here.

$$U = \text{Smaller of} \begin{bmatrix} n_1 n_2 + \dfrac{n_1(n_1+1)}{2} - \Sigma R_1 \\ n_1 n_2 + \dfrac{n_2(n_2+1)}{2} - \Sigma R_2 \end{bmatrix}$$

Now that the U value is known it can be used to compute the z-ratio using the following equation:

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2}{12}(n_1 + n_2 + 1)}}$$

This z-ratio is the normal-approximation test statistic and is used to determine the confidence interval based on the Standard Normal Distribution. The application of a z-ratio is discussed in the next section.

Z-ratio

The z-ratio (a.k.a. z-score) [7] mentioned earlier in both statistical analyses sections applies to the standard normal distribution. This ratio is used to determine a p-value based on standard normal distribution tables found in the back of any statistics book.

A p-value is a measure of level of significance. That means it indicates the probability that the test statistic would lie in the tails of the distribution (in this case, the normal distribution). In other words, you can think of it as an indicator of the probability that the hypothesis is true – i.e., the probability that the means of the probabilistic and Monte Carlo methods are "close together." In simpler terms a low p-value implies low performance of the probabilistic method (since we assume the Monte Carlo method is the "truth"), while a large value close to 1.0 indicates good performance.

In this case rather than worrying about setting a minimum p-value to accept or reject the hypothesis it was decided to use the tests to simply determine what the p-value was. With this p-value, analysis can be done and yet it leaves the reader to form his/her own opinion on the significance of these tests and values.

EXPERIMENTAL SETUP

The first step in determining whether or not the probabilistic algorithm set forth by Hagar is a realistic approach was to determine the actual satellite to station visibilities. In order to do this, MATLAB and the orbital mechanics principals discussed in the previous section were utilized. Satellite possibilities were limited to only LEO satellites in a circular orbit which could only be scheduled once per visibility. This limitation guarantees the ability of the greedy selector to produce an optimal schedule. The reason for the need of an optimal schedule is to determine how close the probabilistic approach comes to the best schedule. The orbit altitude and inclination of the randomly generated satellites used in the analysis can be found below in table 1 and also graphically in figure 6. For simulation data using less than all of the satellites the amount used started from satellite number 1 and continued to the appropriate number. The same is true for the latter station data. Also all satellites where considered to need a contact duration (t_{sat}) of 360 seconds since it falls nicely into the 5 to 7 minute range.

Sat #	Altitude [km]	Inclination [deg]	Sat #	Altitude [km]	Inclination [deg]	Sat #	Altitude [km]	Inclination [deg]
1	587.4	28.1	**21**	956.4	9.2	**41**	387.8	11.3
2	182.1	80.8	**22**	441.7	44.9	**42**	740.1	84.0
3	308.7	78.9	**23**	356.0	1.5	**43**	410.8	2.4
4	974.3	88.5	**24**	494.4	38.0	**44**	275.8	35.8
5	600.8	40.6	**25**	661.0	73.4	**45**	613.0	41.9
6	606.9	59.3	**26**	843.3	1.7	**46**	984.4	40.8
7	847.5	77.3	**27**	829.1	72.3	**47**	197.9	35.5
8	797.4	58.4	**28**	558.3	66.3	**48**	468.8	79.4
9	897.0	18.7	**29**	180.0	39.7	**49**	203.6	51.1
10	836.0	11.2	**30**	448.8	52.4	**50**	627.7	61.0
11	682.7	7.4	**31**	441.8	10.6	**51**	723.7	27.0
12	569.7	66.3	**32**	184.2	16.8	**52**	492.1	76.3
13	883.0	10.1	**33**	977.6	56.7	**53**	483.6	20.7
14	945.3	38.7	**34**	840.7	51.2	**54**	544.9	78.3
15	982.3	12.6	**35**	362.9	25.1	**55**	344.8	22.5
16	768.1	76.6	**36**	289.1	34.4	**56**	690.4	37.2
17	478.6	55.0	**37**	318.5	22.1	**57**	733.3	26.2
18	576.8	26.4	**38**	769.6	38.1	**58**	451.8	9.8
19	263.8	13.7	**39**	899.6	17.5	**59**	477.1	66.5
20	223.5	40.6	**40**	853.0	0.4	**60**	859.8	40.0

Table 1 - Satellite Data

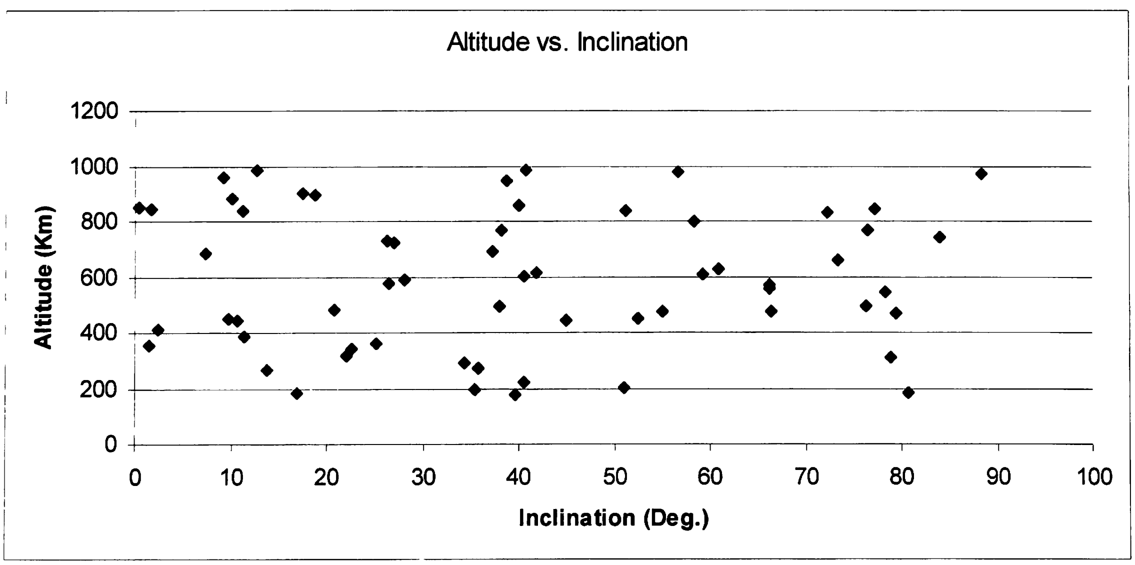

Figure 6 - Scatter Plot of Satellite Data

Along with the satellite data, data about the stations is also necessary. Two sets of stations were used and results from both were collected and used to investigate the ability of the algorithm. One set is the "Global" set, which consists of stations spread all around the globe. The second set is the "Western Hemisphere" set which contains only stations spread throughout the western hemisphere, all of these are in the U.S. except for the Antigua station. The following tables contain all the necessary latitude and longitude information about the stations used in the simulation. The tables are followed by there respective scatter plots showing the spread of stations around the world.

Location	Stn #	Latitude [deg]	Longitude [deg]
Floating Station	1	0.00	0.00
Maui, HI	2	20.71	-156.26
Cape Cod, MA	3	41.75	-70.54
Ascension, Atlantic	4	-7.91	-14.40
Kourovka, Russia	5	57.01	59.58

Table 2 - Global Station Data

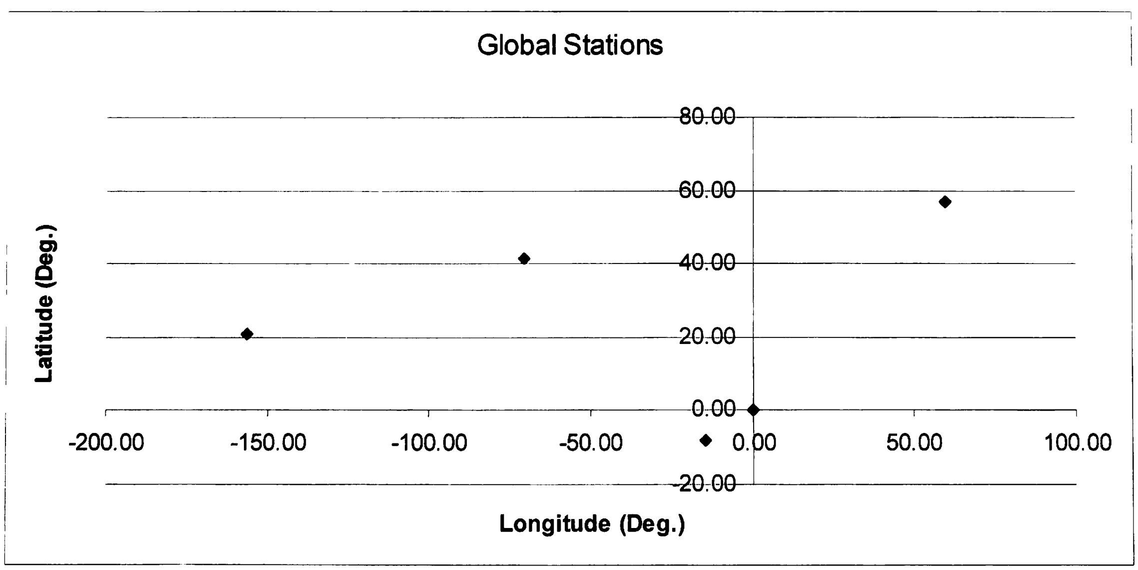

Figure 7 - Global Station Scatter Plot

Location	Stn #	Latitude [deg]	Longitude [deg]
Floating Station.	1	0.00	0.00
Clear, AK	2	64.29	-149.19
Cavalier, ND	3	48.72	-97.90
Maui, HI	4	20.71	-156.26
Antigua, Caribbean	5	17.14	-61.79

Table 3 - Western Hemisphere Station Data

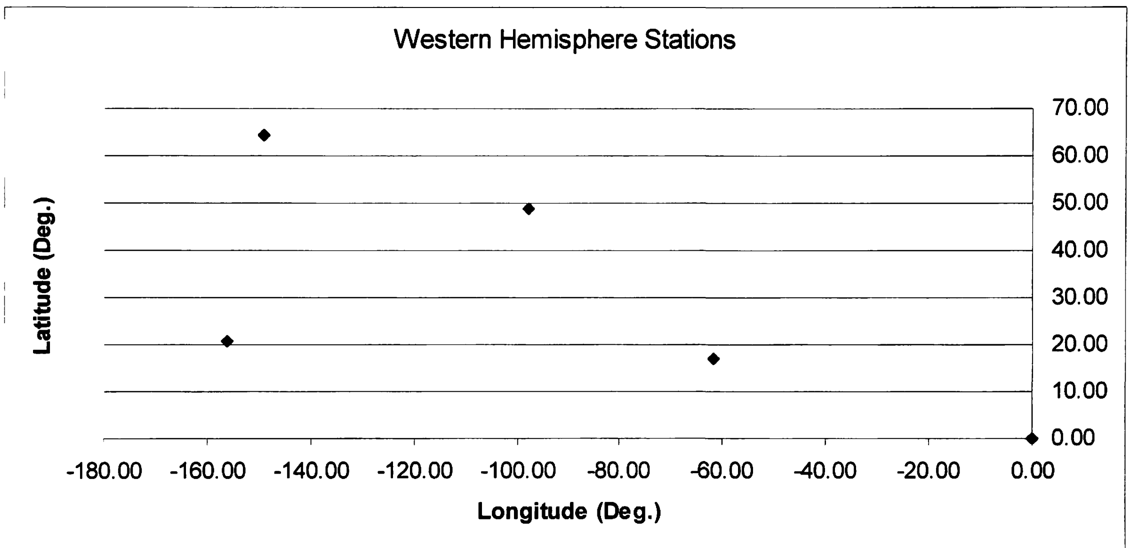

Figure 8 - Western Hemisphere Station Scatter Plot

The MATLAB program was then implemented running combinations involving 2, 3, 4 and 5 stations and 10, 20, 30, 40, 50 and 60 satellites. All combinations were run for each Global and Western Hemisphere station set. Also a set where satellites required 6 contacts per day and a set where the satellites required 10 contacts per day were run for all combinations for comparison. Each combination was run for 25 iterations and the results were accumulated and averaged for the 25 runs. The reason 25 was chosen for the number of runs was because the results were found to remain nearly unchanged for any larger sample. All results were recorded and appear in the next section. The following sequence of steps shows a basic order of operations for the program written.

- *Obtain Visibility Times*

For Iterations = 1 to 25

For Satellite i = 1 to n

For time t = 0 to t_{final} in increments of Δt

Calculate orbit state (position and velocity) for time t

For Station j = 1 to m

If satellite i crosses FOV view of station j then

1) Increment count of visibilities (contact opportunities).

2) Save satellite-station FOV entry and exit times

End

End

End

- *Next Schedule Satellite Visibilities*

Arrange the satellite contact visibilities in a list by ascending order of visibility end times.

Step through list; at each step, "schedule" (count) a contact only if the begin time of the visibility is later than the preceding end time.

End (25 Iterations)

- *Calculate metrics*

Feasibility Ratio

Utilization Ratio

Performance Ratio

RESULTS

The five metrics discussed earlier were compiled and are all included in this section. One set of values (Monte Carlo and Probabilistic) is included for each metric; the remaining data has been placed in Appendix G as it has the same trends as the data shown here. Also a 3-D surface plot follows the data sets for a visualization of the trends involved. These results are simply shown here, any interpretation or analysis of these metrics are left to the Analysis section.

The following tables contain the Monte Carlo results from the MATLAB program written (Appendix B).

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	113.9	157.2	219.5	261.0
20	252.3	327.5	468.2	535.6
30	372.6	485.1	693.0	793.0
40	516.5	654.5	939.0	1056.4
50	617.2	793.2	1134.5	1282.3
60	752.1	962.3	1376.7	1554.8

Table 4 - Opportunities, Global Stations, Monte Carlo, 6/Day

	# of Scheduled Contacts (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	53.6	57.3	58.4	59.9
20	104.5	116.7	118.1	119.8
30	139.5	173.5	177.5	179.6
40	164.4	219.4	236.4	239.1
50	180.2	250.3	293.8	297.7
60	194.7	273.6	340.3	356.1

Table 5 - Scheduled Contacts, Global Stations, Monte Carlo, 6/Day

	Feasibility Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	1.898667	2.619778	3.659111	4.350000
20	2.102444	2.728778	3.901778	4.463111
30	2.069778	2.694889	3.850000	4.405333
40	2.152222	2.726944	3.912333	4.401722
50	2.057422	2.643956	3.781600	4.274444
60	2.089037	2.673185	3.824185	4.318815

Table 6 - Feasibility Ratio, Global Stations, Monte Carlo, 6/Day

	Utilization Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.470154	0.364662	0.266003	0.229527
20	0.414015	0.356244	0.252136	0.223710
30	0.374383	0.357632	0.256065	0.226460
40	0.318198	0.335174	0.251782	0.226288
50	0.291994	0.315537	0.258985	0.232181
60	0.258936	0.284291	0.247160	0.229032

Table 7 - Utilization Ratio, Global Stations, Monte Carlo, 6/Day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.892667	0.955333	0.973333	0.998444
20	0.870444	0.972111	0.983778	0.998444
30	0.774889	0.963778	0.985852	0.997630
40	0.684833	0.914000	0.985056	0.996056
50	0.600756	0.834267	0.979378	0.992444
60	0.540926	0.759963	0.945185	0.989148

Table 8 - Performance Ratio, Global Stations, Monte Carlo, 6/Day

The values of the previous tables are to be compared with results from the probabilistic method. The values which came from the probabilistic method can be found in the following tables which contain the same types of information as the previous tables.

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	120.04	165.47	230.54	272.86
20	267.18	345.51	494.59	563.84
30	395.16	512.55	732.76	835.57
40	548.81	693.20	996.32	1115.78
50	655.28	838.50	1201.02	1352.83
60	797.43	1016.55	1457.99	1639.92

Table 9 - Opportunities, Global Stations, Probabilistic, 6/Day

	# of Scheduled Contacts (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	52.12	52.17	53.97	56.96
20	100.17	110.69	113.72	116.41
30	135.66	163.86	171.79	174.39
40	173.90	213.85	230.08	233.27
50	195.40	254.97	282.99	288.80
60	222.27	299.60	337.56	345.21

Table 10 - Scheduled Contacts, Global Stations, Probabilistic, 6/Day

	Feasibility Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	2.000593	2.757873	3.842317	4.547731
20	2.226540	2.879212	4.121583	4.698630
30	2.195359	2.847504	4.070862	4.642029
40	2.286729	2.888324	4.151342	4.649080
50	2.184254	2.794999	4.003405	4.509439
60	2.221251	2.831602	4.061253	4.568011

Table 11 - Feasibility Ratio, Global Stations, Probabilistic, 6/Day

	Utilization Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.434245	0.315253	0.234110	0.208743
20	0.374925	0.320367	0.229933	0.206456
30	0.343288	0.319686	0.234440	0.208708
40	0.316859	0.308501	0.230928	0.209062
50	0.298198	0.304075	0.235625	0.213476
60	0.278732	0.294724	0.231528	0.210507

Table 12 - Utilization Ratio, Global Stations, Probabilistic, 6/Day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.868748	0.869429	0.899526	0.949309
20	0.834785	0.922404	0.947687	0.970059
30	0.753641	0.910308	0.954374	0.968829
40	0.724571	0.891052	0.958662	0.971948
50	0.651340	0.849890	0.943304	0.962657
60	0.619134	0.834540	0.940292	0.961598

Table 13 - Performance Ratio, Global Stations, Probabilistic, 6/Day

The following plots are of the data from the previous tables. The Monte Carlo information is placed next to the corresponding probabilistic data for ease of comparison.

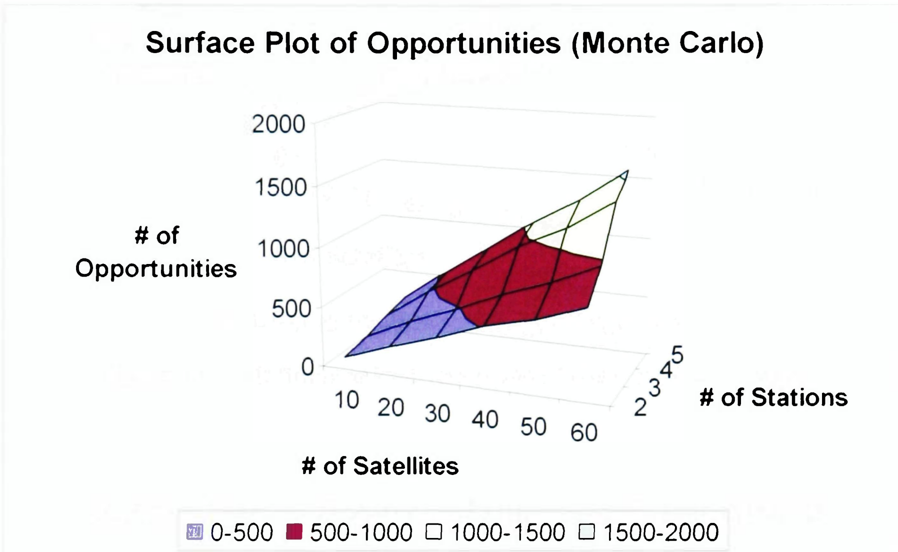

Figure 9 - 3-D Surface Plot, Opportunities, Monte Carlo

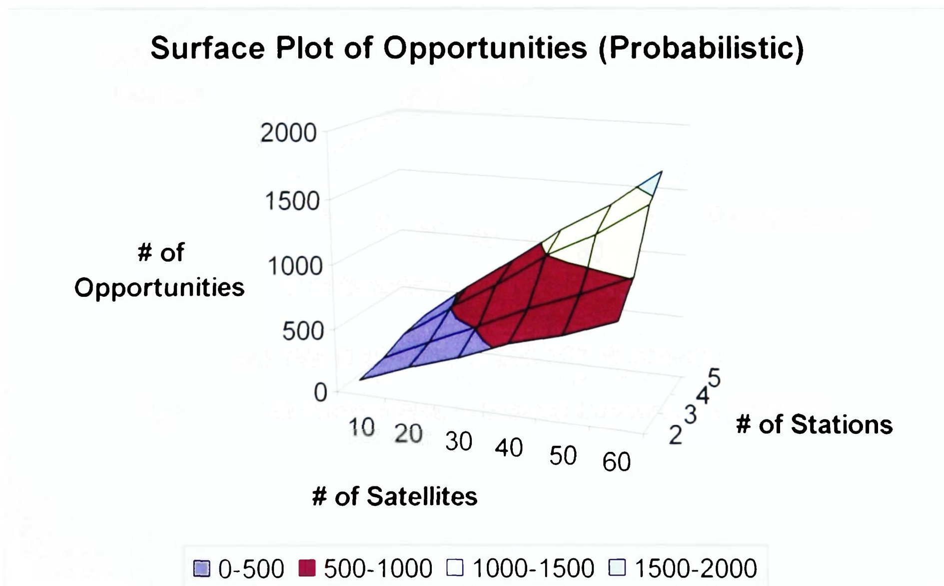

Figure 10 - 3-D Surface Plot, Opportunities, Probabilistic

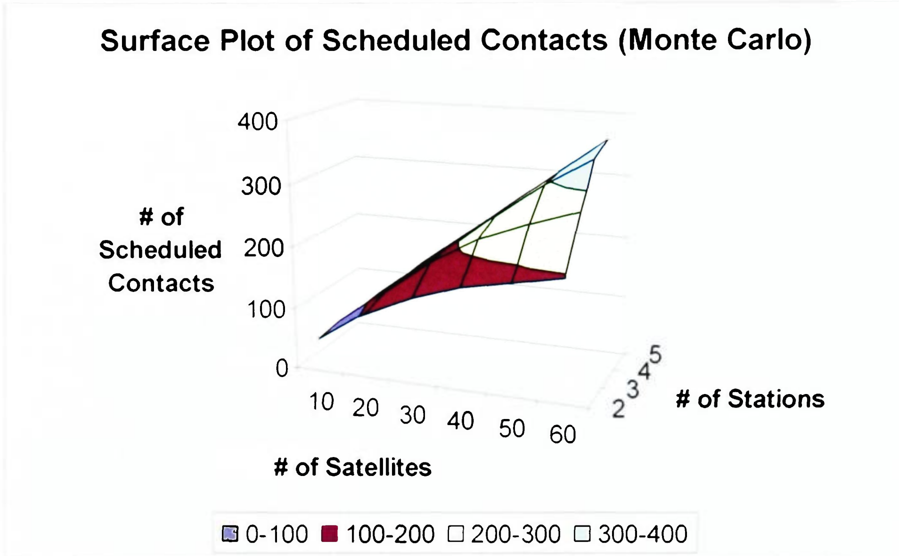

Figure 11 - 3-D Surface Plot, Scheduled Contacts, Monte Carlo

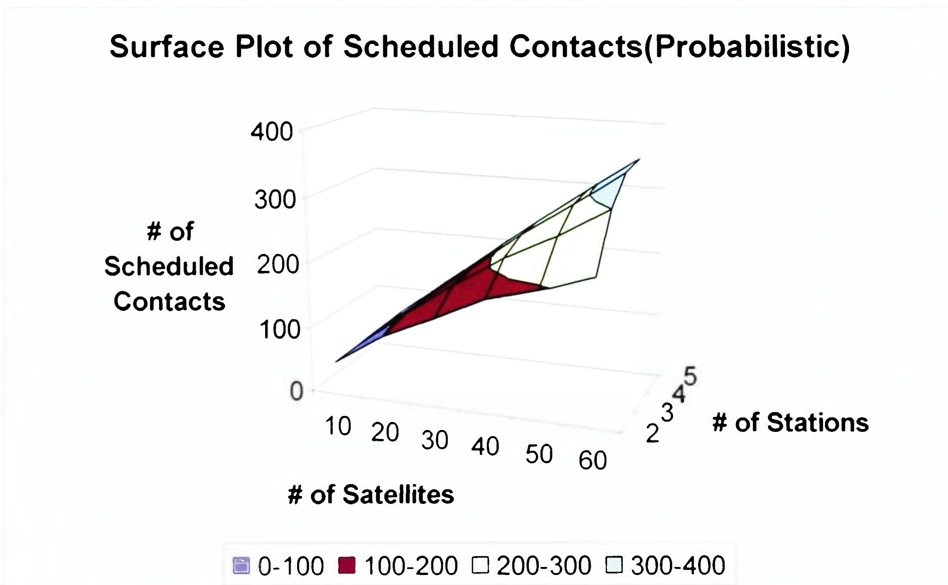

Figure 12 - 3-D Surface Plot, Scheduled Contacts, Probabilistic

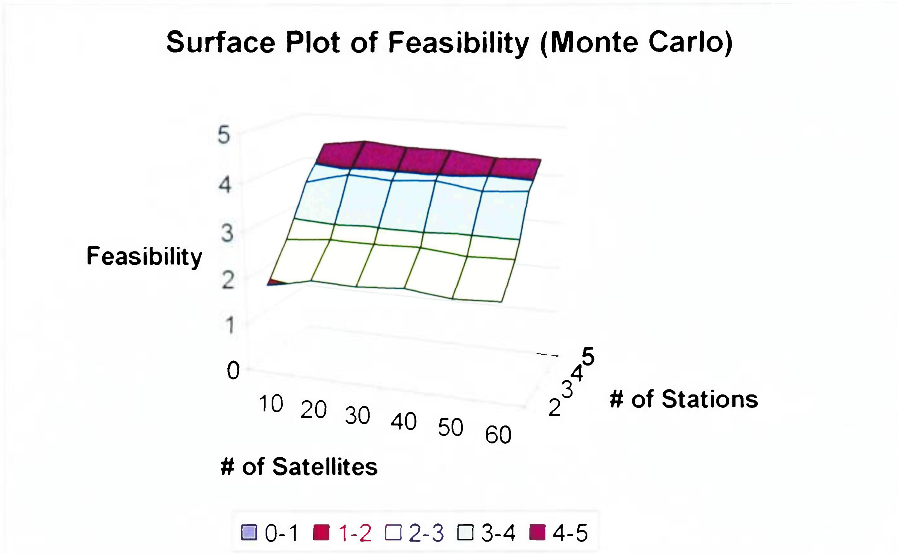

Figure 13 - 3-D Surface Plot, Feasibility Ratio, Monte Carlo

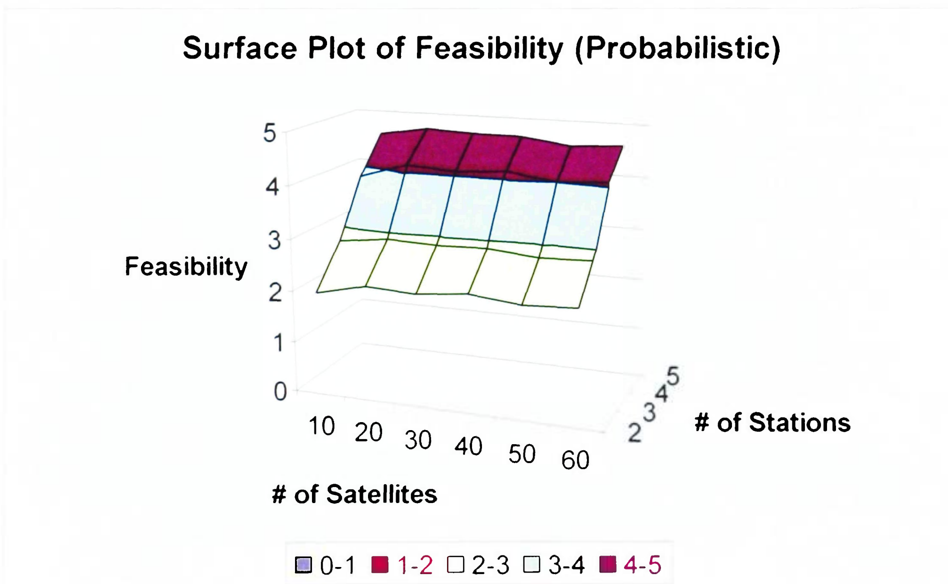

Figure 14 - 3-D Surface Plot, Feasibility Ratio, Probabilistic

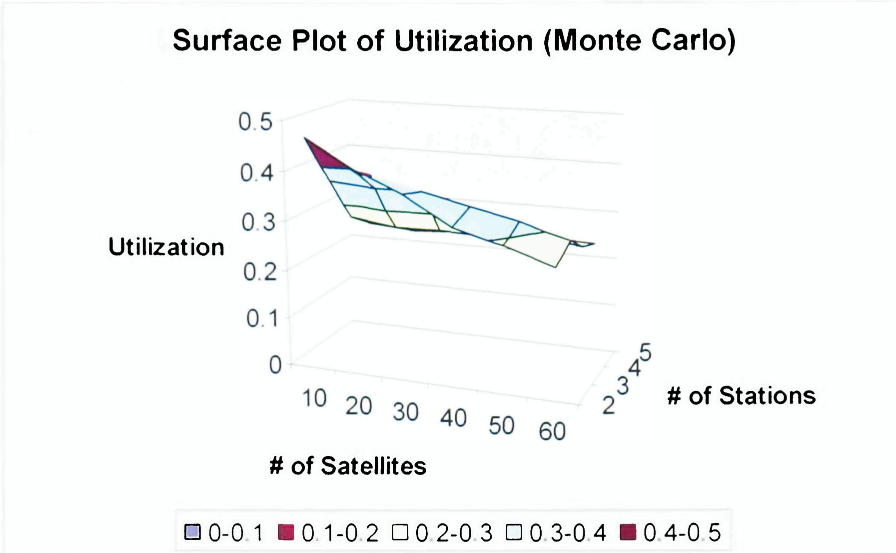

Figure 15 - 3-D Surface Plot, Utilization Ratio, Monte Carlo

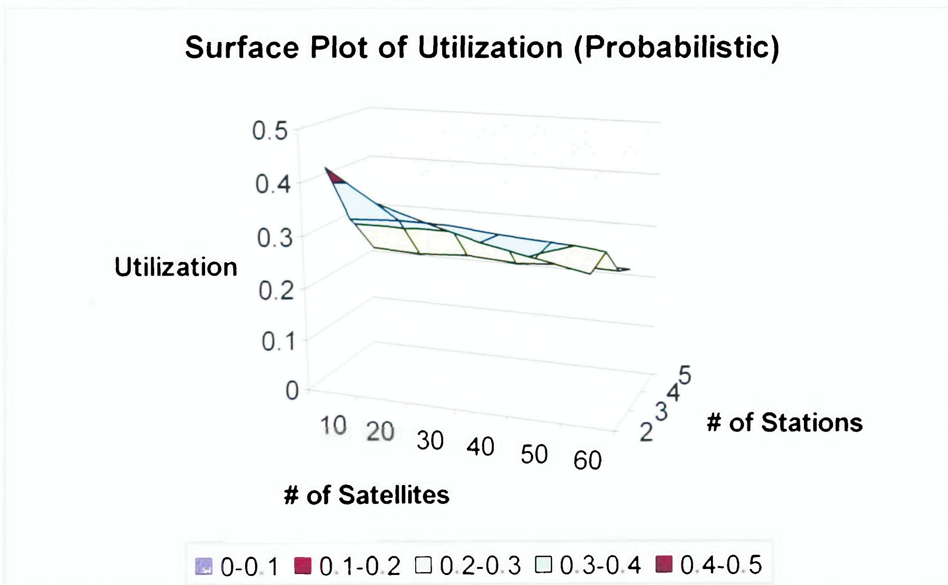

Figure 16 - 3-D Surface Plot, Utilization Ratio, Probabilistic

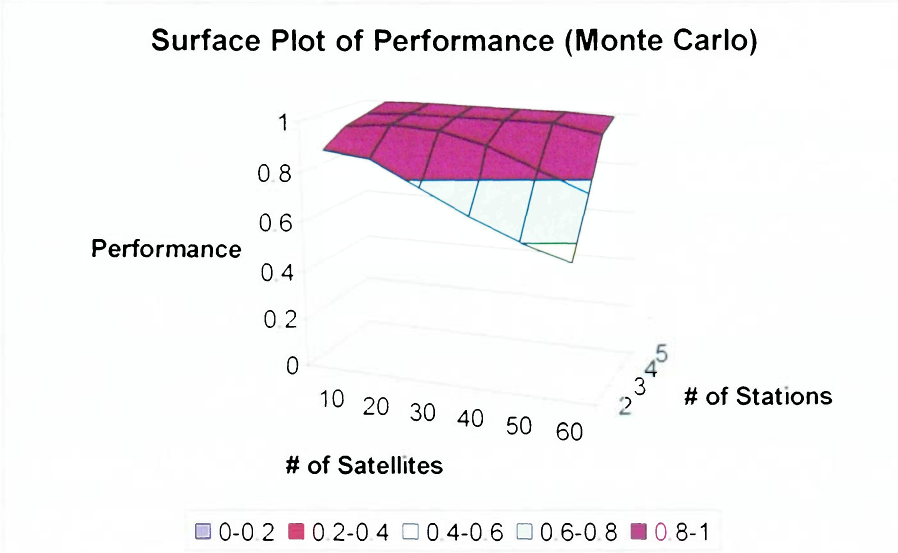

Figure 17 - 3-D Surface Plot, Performance Ratio, Monte Carlo

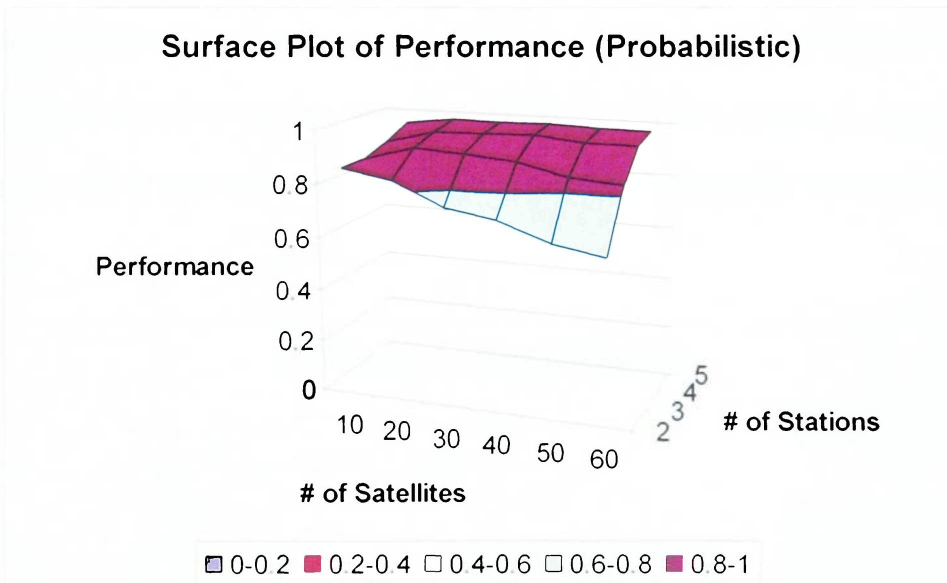

Figure 18 - 3-D Surface Plot, Performance Ratio, Probabilistic

An analysis of the previous results listed appears in the following section. Note that the previous results did not come from the respective programs in this form. The necessary results were compiled in Excel and presented here. Appendices E and F contain examples of the raw MATLAB and probabilistic results respectively.

ANALYSIS

Once the values from the previous calculations were obtained the next step was to do some numerical comparisons. The following plots display percent differences. The results of the Wilcoxon signed ranks tests follow.

The first set of analysis data which will be considered is the percent difference between the probabilistic method and the Monte Carlo simulations. A single percent difference plot for each metric follows (Global stations, 6 contacts/day). The rest of the percent difference plots are contained in Appendix G due to their similarity to the plots which follow. If a specific value for a case is desired please refer also to Appendix G following the plots. Note that the percent differences for the number of opportunities and the number of scheduled contacts were allowed to have negative values. A negative occurring in one of these sets indicates that the probabilistic approach "over-estimated" the value.

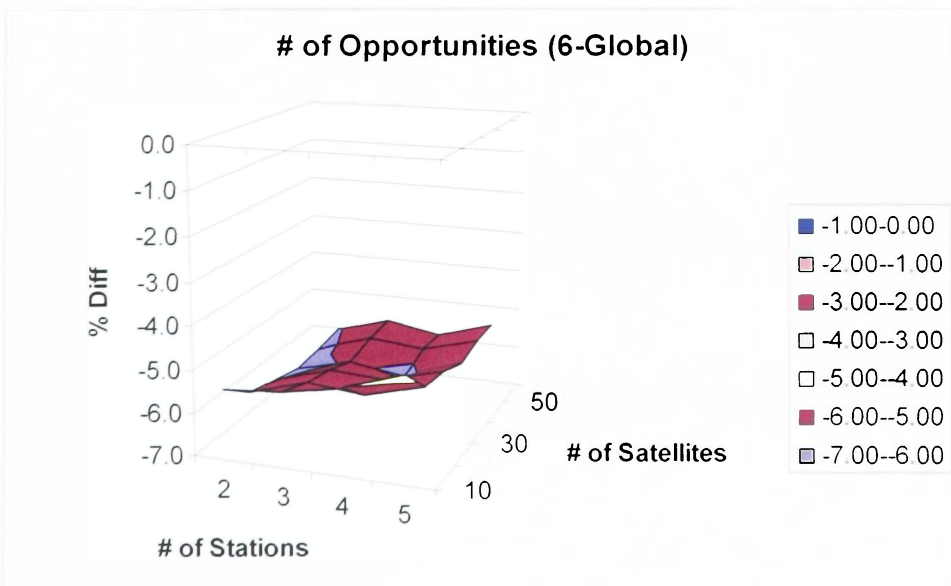

Figure 19 - Percent Difference, Opportunities, Global Stations, 6/Day

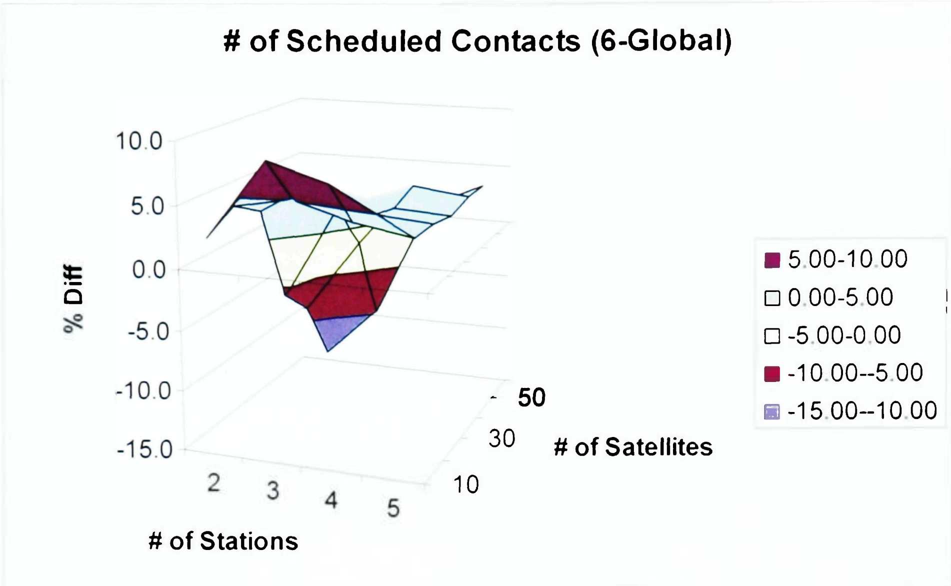

Figure 20 - Percent Difference, Scheduled Contacts, Global Stations, 6/Day

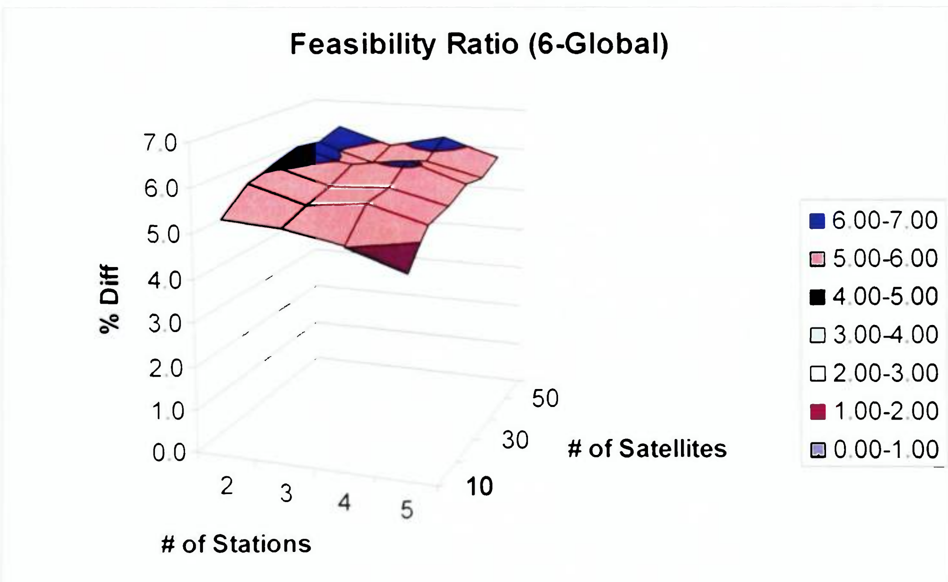

Figure 21 - Percent Difference, Feasibility, Global Stations, 6/Day

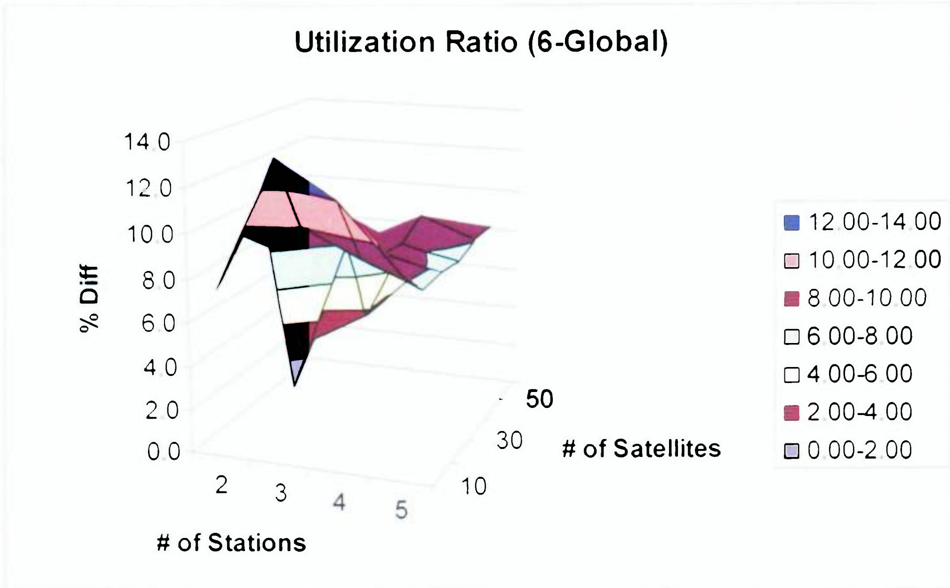

Figure 22 - Percent Difference, Utilization, Global Stations, 6/Day

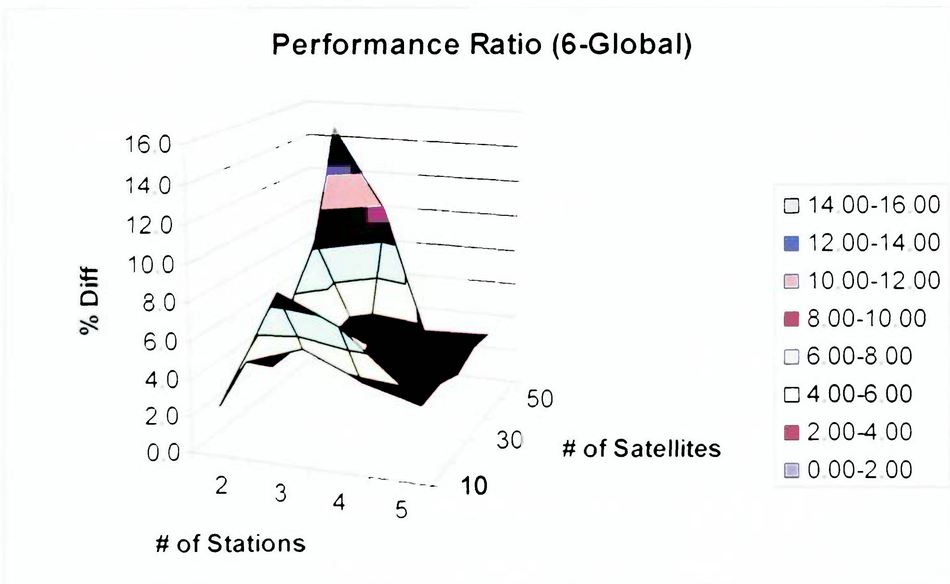

Figure 23 - Percent Difference, Performance, Global Stations, 6/Day

The first comparison of interest is the number of opportunities calculated by both methods. The percent difference seems to be reasonably and consistently falling below

10% and most of the time hovering around the plus or minus 5% area. Consider the fact that actual values being compared generally are in the range of 100-1500 opportunities. This means that the differences are only around 5 opportunities in the low portion of the range and only up to roughly 75 in the high portion. Taking into account that the 1500 opportunity area of consideration involves 60 satellites it is known that this is a discrepancy of (on average) much less than 2 opportunities per satellite. This discrepancy suggests, in terms of opportunity calculations, the two methods are similar. Since the Feasibility Ratio is based directly on the number of opportunities calculation it goes without saying the percent differences of the two will be very similar. One should also note the negative percent differences in these categories indicate an over-estimation of opportunities. A conservative value (under-estimation) would be preferable.

Looking at the scheduled contacts percent differences is a slightly different story. A much larger spread of percent differences start the graphs and taper into a seemingly reasonable value. Remember that the utilization ratio and the performance ratio are calculated using the number of scheduled contacts. The influence of the discrepancy in scheduled contacts in the two related ratios is immediately clear, all three start with large fluctuations in percent differences and taper close to a common value as the number of stations rises. Note that there are a number of situations where an over-estimation occurs; again a more conservative value would be desirable.

While looking through all of the percent difference graphs an obvious trend seems to appear. As the number of stations goes up the differences in methods seems to diminish. Initially this seems to indicate that as long as there are a large number of stations involved the probabilistic approach may hold some validity. Upon further inspection one realizes the situations involving a large number of stations are not the ones of utmost interest here. The probabilistic method is being tested so it can be used to determine if an existing network structure can handle another satellite. If the network under consideration already has a large number of stations and isn't being stressed to begin with, it probably doesn't need to be assessed. Any network that shuts down "extra" stations to conserve money immediately falls closer to being saturated. This means the area of most interest for this verification would be cases where the network is close to being saturated (feasibility ratio close to 1.0).

The feasibility ratios are always closer to one for lower numbers of stations which intuitively makes sense. Fewer stations equal fewer opportunities and demands more from a network. Taking this fact into account the ability of the probabilistic method comes into question based on the percent differences of all categories. Even for the lower numbers of satellites (less stressed situations) percent differences can range from 35 percent down to less than 5 percent.

If the percent differences indicate that the probabilistic method is flawed can anything be salvaged from the work done on it? In order to determine if the distributions of both sets of data are analogous a Wilcoxon signed-ranks test and a Mann-Whitney U test were performed. Only the sets of opportunities and scheduled contacts were compared using these methods. The reason for using only these data was that comparing ratios using probabilistic tests generally tend to add an element of error. These tests both result in z-ratios due to normal-approximation procedures because of the large sets of data meaning a standard normal distribution table was used to interpret the data.

The Wilcoxon signed-ranks test was applied first. As mentioned earlier the Wilcoxon test results in a z-ratio. The z-ratios are included in table 14 with the related p-value.[7]

	Wilcoxon Signed-Ranks Test			
	# of Opportunities		# of Scheduled Contacts	
	Global, 6/day	Global, 10/day	Global, 6/day	Global, 10/day
z-ratio	4.28	4.28	1.54	2.56
p-value	0.0	0.0	0.1236	0.0105
	Western, 6/day	Western, 10/day	Western, 6/day	Western, 10/day
z-ratio	4.28	4.28	0.39	3.22
p-value	0.0	0.0	0.6965	0.0013

Table 14 - Wilcoxon Signed-Ranks Test Results

The p-values from this inspection tend to indicate that there is no consistent correlation between the probabilistic method's calculation of opportunities and the Monte Carlo's calculation. This would suggest that the probabilistic approach cannot accurately calculate the number of opportunities for a given situation. There is also a very poor correlation for the scheduled contacts. Two of the p-values in the scheduled contacts are near 0 with one other around 0.1. The best value is the Western Hemisphere case for 6 contacts per day, which is around 0.70. 0.70 is still, in most cases, of questionable value for any scientific research and indicates that there is a lack of ability for the scheduling

[7] The p-value as used here is actually $1-p$, where p, is the traditional significance level. Thus a traditional value of $p=0.05$ corresponds here to a value of $p=0.95$. For the traditional p-values, subtract the values shown in the tables from 1.

portion of the probabilistic method. In general the Wilcoxon test implies that there is very little merit to the probabilistic method.

As a comparing method the Mann-Whitney U test was applied next for the same sets of data and again gave a z-ratio. The z-ratios were used to find p-values and both are include in table 15.

	Mann-Whitney U Test			
	# of Opportunities		# of Scheduled Contacts	
	Global, 6/day	Global, 10/day	Global, 6/day	Global, 10/day
z-ratio	0.43	0.43	0.29	0.54
p-value	0.6672	0.6672	0.7718	0.5892
	Western, 6/day	Western, 10/day	Western, 6/day	Western, 10/day
z-ratio	0.43	0.43	0.06	0.37
p-value	0.6672	0.6672	0.9522	0.7114

Table 15 - Results Mann-Whitney U Test

The results of this particular test seem to contradict the results of the Wilcoxon test. Again the number of opportunities tests all have identical values but are much closer to 1.0 than before. Remember that a p-value of 1.0 indicates the means of the populations are equal. While they are closer, 0.67 is still a questionable value. The scheduled contacts tests seem to show a stronger correlation than the opportunities tests with three values above 0.70. These results seem to indicate promise in the probabilistic method but not complete acceptance.

What can be taken from these tests? First the calculation of opportunities per day should be considered because without a proper value of opportunities it is impossible to know if scenarios are even feasible. The Wilcoxon results indicate the opportunity calculation is seriously flawed, the Mann-Whitney results indicate there is a decent correlation and the percent differences seem to also indicate a good correlation. Looking at all three tests the feeling of this investigator is that the probabilistic approach is sufficient in the area of estimating opportunities but not great. This means that the opportunities calculation will give a good estimate but a certain amount of error will always be present.

The next category is scheduled contacts. For scheduled contacts the Mann-Whitney test seems to indicate a decent correlation between methods. The percent differences calculations seem to indicate that while there are instances the probabilistic method gets the number of scheduled contacts right it often seems to miscalculate them in "over-stressed" network situations. Finally the Wilcoxon test suggests there is little correlation between methods. Looking at all three tests seem to suggest there is some merit to the scheduling portion of the algorithm but not enough to deem it usable in any case. More likely the scheduling portion of the probabilistic method should be adjusted and retested.

CONCLUSIONS & RECOMMENDATIONS

The ability of the probabilistic approach to capture the nature of the scheduling problem seems to be questionable. The overall conclusions are as follows:

1. **The probabilistic method works sufficiently for estimating the number of scheduling opportunities a group of satellites will have with a given station network.**
2. **The probabilistic method estimates the number of scheduled contacts well in "under-stressed" situations but poorly in "over-stressed" situations.**
3. **Over-estimated values of contact opportunities occur frequently and are undesirable due to the fact it indicates the network has more ability than it actually does.**
4. **Overall the method shows promise but needs more development before use in industry.**

The recommendations that come from this work are as follows:

1. **Make adjustments to the probabilistic method and do more testing. E.g., the empirical factor for adjusting the scheduling probability is not on a firm theoretical basis. More attention to a theoretical basis could help improve performance.**
2. **Check for factors involved with the scheduling probability or other aspects of the probabilistic method which are possibly being overlooked that will improve the methods ability. (Perhaps something which would help the method under-estimate ability.)**

3. Look at the assumptions Hagar involved in the probabilistic method and make sure they aren't over simplifying the scheduling problem where it is unnecessary. E.g., Modeling the location of the orbit plane and the satellite's location as independent events, although they aren't.

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Appendix A

Probabilistic Methodology for Predicting Satellite Tracking Resources

The material in this appendix was extracted from ERAU-funded research work done during summer, 2005, and published as Reference 1.

Methodology

Initial Assumptions. The initial approach to solving the tracking resource prediction problem entails certain simplifying assumptions:

1. The initial assumption of low altitude (180 to 1000 km), circular satellite, Keplerian orbits only. This excludes the extensive class of high altitude and/or elliptic orbits, as well as perturbation effects. However, it greatly simplifies this initial analysis, and is essential for the testing and evaluation of the approach (which is based on Monte Carlo simulation of the Burrowbridge optimization algorithm).
2. Independence between satellite ephemeredes (orbital elements, and launch dates and locations).
3. Performance depends only on tracking station latitude, and does not depend upon tracking station longitude. This is a rather drastic assumption; however for this initial research and development it simplifies the analysis.

While these may appear perhaps overly restrictive, they fall within the category of "learn to walk before you run."

Kinematics. Satellite communications contacts require knowing when each satellite is in view of a tracking facility. The process of determining this is completely known and analytic. The key difficulty is determining an analytic or semi-analytic methodology for predicting *when* these visibility periods occur without resorting to extensive computer calculation. Components of such an analytical representation are important for incorporating the appropriate probability distributions representing above-mentioned uncertainties.

Statistics. To accommodate the statistics associated with the inherent uncertainties, probability distributions associated with the likelihood of multiple satellites competing for contact with tracking stations are developed. These also entail modeling the distributions of the satellite orbit element and contact uncertainties.

Development

Probability of Conflict.

Assume that satellite k is over station j and that ia contact is to be planned between the satellite and the station. With no knowledge of the actual ephemeris of any other satellites that might possibly need a contact from that same station, there is nevertheless some probability that at least one other satellite may be over the sation at the same time and also require a contact. This constitutes the "conflict" or contention for the contact.

To address this issue we need to consider not only the probability associated with a competing (k^{th}) satellite's RAAN being within the longitude band required for a station pass, but also the probability $p_{\theta kj}$ that competing satellite, k, is over station j at the same time as the satellite under consideration (satellite i). The probability $p_{\theta kj}$ is actually a conditional probability, conditioned on the event that satellite k's RAAN is in the same longitude band required for a pass over station j. Therefore, the probability that satellite k is over station j at the same time as satellite i is the product $p_{\Lambda kj} p_{\theta kj}$, which follows since we assume that the argument of latitude is independent of the RAAN.[8]

This is not the whole story, however. In addition we need to know whether or not a contact by satellite k, is to be attempted. One modeling approach is to assume that the distribution of contact requests is uniform; that is, that a contact at one opportunity is just as likely at any other opportunity. Hence if there are a total of n_{Rk} required contacts for satellite k, and n_{Ok} total opportunities, then the probability of requesting a contact at that simultaneous time can be written as

$$p_{ck} = \frac{n_{Rk}}{N_{Ok}} = \frac{n_{Rk}}{\sum_{j=1}^{m} n_{Okj}}$$

where $N_{Ok} = \sum_{j=1}^{m} n_{Okj}$ is the sum of the total daily opportunities for satellite k at station j.

Then the probability of conflict - that satellite k will compete for a contact at the same station and time as satellite i is therefore

$$\boxed{p_{ijk}^{\times} = p_{\Omega kj} p_{\theta kj} p_{ck}}.$$

Further, the probability that satellite k is *not* in conflict is simply

$$\boxed{p_{ijk}^{\bar{\times}} = 1 - p_{\Omega kj} p_{\theta kj} p_{ck}}$$

This allows us to compute the corresponding probabilities for all other satellites:

The probability that *one or more* other satellites, k, are in conflict with satellite i follows from elementary probability theory:

$$\boxed{p_{ij}^{\times} = 1 - \prod_{k=1}^{n}{}' \bar{p}_{kj} = 1 - \prod_{k=1}^{n}{}' \left(1 - p_{kj}\right)}$$

where the prime $(\ )'$ means the product is taken over all k except $k = i$. Similarly, the probability of *no conflict* with any other ssatellite at station j is

[8] That argument of latitude and RAAN may be assumed independent follows from the fact that the launch date is not known accurately enough into the future. While knowledge of launch date is sufficient to determine both RAAN and argument of latitude, the relationships for mapping such uncertainty into the future are complicated at best. Absent such knowledge, and in the interests of simplicity, it is sufficient to assume independence.

$$\boxed{p_{ij}^{\bar{x}} = \prod_{k=1}^{n} \bar{p}_{kj}{}' = \prod_{k=1}^{n} \left(1 - p_{kj}\right)'}$$

Probability of Being Over the Same Station. We have yet to describe the probability, $p_{\theta kj}$, which is the probability that satellite k passes within tracking range of station j, given that k's RAAN is in the longitude band for a pass at station j. Here we make an approximation, since the precise solution is at best complicated, and at worst, unsolvable. Assume the coverage region is actually on a plane instead of a sphere. Imagine a plane cutting through the Earth's surface, intersecting the station's effective horizon. Figure 24 below shows this plane with a typical satellite path as it approaches the coverage region. The radius, r, is the radius of the coverage region in this planar representation.

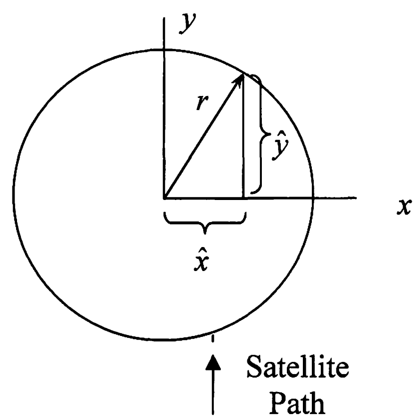

Figure 24 - Mean Tracking Pass Length

We are interested in where, along the x-axis, the satellite's path penetrates, and, correspondingly, the value of the path, $\hat{y}$. If we know the expected value, $\hat{y}$, then by symmetry $2\hat{y}$ approximates the average path which the satellite follows through the coverage region. We can convert this distance to an angular measure, $2\hat{\alpha}$. Then forming the quotient, $\frac{\hat{\alpha}}{\pi}$, provides the probability that the satellite is within the coverage region at any time, given that the RAAN is within the longitude band required for a pass. The trick then is to determine the expected value of y.

Assume that the point where the satellite path crosses the x-axis is uniformly distributed over the interval $\left[-r, r\right]$. Because of symmetry, we can limit our consideration to the interval $\left[0, r\right]$. This gives us the corresponding probability distribution for x as $F_X = \frac{x}{r}$.

Substituting for x yields the probability distribution for Y:

$F_Y = F_X\left(x = \sqrt{r^2 - y^2}\right) = \frac{\sqrt{r^2 - y^2}}{r}$. The corresponding density function for Y is, by differentiation, $f_Y = \frac{d}{dy}\left(\frac{\sqrt{r^2-y^2}}{r}\right) = \frac{1}{2r}\frac{-2y}{\sqrt{r^2-y^2}} = -\frac{y}{r\sqrt{r^2-y^2}}$. With this we can find the expected value of y:

$$\hat{y} = E(Y) = \int y\, f_Y(y)\, dy = \int_r^0 -\frac{y^2}{r\sqrt{r^2-y^2}}\, dy = r\frac{\pi}{4}$$

However, since we are interested in the complete path, we multiply by 2 to get

$$2\hat{y} = r\frac{\pi}{2}.$$

We'd like to convert $\hat{y}$ to an angle, $\hat{\alpha}$, as shown in figure 25, below.

$$\hat{\alpha} = \sin^{-1}\left(\frac{\hat{y}}{R_e}\right) = \sin^{-1}\left(\frac{r\pi}{4R_e}\right).$$

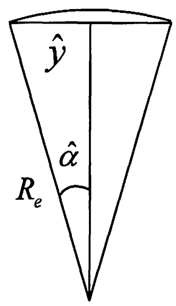

Figure 25 - Mean Pass Arc Length

Now, we don't really know what r is, but we do know the angular extent of the half field of view, $\frac{w_0}{2}$, which is the angle corresponding to r. Then $r = R_e \sin\frac{w_0}{2}$, and therefore

$$\hat{\alpha} = \sin^{-1}\left(\frac{\hat{y}}{R_e}\right) = \sin^{-1}\left(\frac{R_e \sin\left(\frac{w_0}{2}\right)\pi}{4R_e}\right) = \sin^{-1}\left[\frac{\pi}{4}\sin\left(\frac{w_0}{2}\right)\right]$$

For example, at 1000 km altitude, the extent of the coverage region is

$$\frac{w_0}{2} \approx 29° \Rightarrow \hat{\alpha} = \sin^{-1}\left[\frac{\pi}{4}\sin(29°)\right] = 22.38°.$$

Then finally, the probability of satellite k being over the station, given that the RAAN is within the required longitude band, is

$$\boxed{p_{\theta kj} = \frac{\hat{\alpha}}{\pi} = \frac{\sin^{-1}\left[\frac{\pi}{4}\sin\left(\frac{w_0}{2}\right)\right]}{\pi}}.$$

Probability of Scheduling Required Contacts. The probability $\boxed{p_{ij}^{\bar{x}} = \prod_{k=1}^{n} \bar{p}_{kj}{}' = \prod_{k=1}^{n} \left(1-p_{kj}\right)'}$ is the probability that at a particular satellite has a conflict-free opportunity to schedule a contact at station j, given that it is within tracking range of the station. (While we do not know if the contact would be requested/scheduled, we do know the likelihood based on the estimate of the probability, $p_{ci} = \frac{n_{Ri}}{N_{Oi}} = \frac{n_{Ri}}{\sum_{j=1}^{m} n_{Oij}}$ discussed above.) We are really interested in whether or not, n_{Ri}, the total number of contacts required for satellite i, could be scheduled conflict-free on the network (across all $j = 1, 2, \ldots, m$ stations). During a 24 hour period, there are $N_{Oi} = \sum_{j=1}^{m} n_{Oij}$ opportunities available over the m tracking stations in the network. If we assume that the number of opportunities at each station is the same, then there are $\binom{N_{Oi}}{n_{Ri}}$ ways to schedule the n_{Ri} opportunities for satellite i on the network of m stations.

Basic Algorithm for Probability of Satellite Scheduling:
Probability of a single configuration for satellite i:

$$\underline{p}_{si}\left(n_{Ri}, n_{Oij}, m, p_{ij}^{\bar{x}}\right) = \sum_{x_1=0}^{n_{ri}} \sum_{x_2=0}^{n_{Ri}-x_1} \sum_{x_3=0}^{n_{Ri}-x_1-x_2} \cdots \sum_{x_{m-1}=0}^{n_{Ri}-x_1-x_2-\ldots-x_{m-2}} \prod_{j=1}^{m} \binom{n_{Oij}}{x_j} p_{sij}^{x_j} q_{sij}^{n_{Oij}-x_j} \; ; \; q_{ij} \square 1 - p_{ij}$$

(For simplicity, we will use the shorthand $p_{sij} \equiv p_{sij}^{\bar{x}}$.) Note that here we use $p_{ij} \equiv p_{ij}^{\bar{x}}$, the probability of no conflict, as the probability of satellite i being scheduled at station j. This equation is subject to the important constraints:

$$\sum_{j=1}^{m} x_j = n_{Ri} \;, x_j \le n_{Oij}, \text{ and } \binom{n}{k} \equiv 0 \text{ for } k > n.$$

For the complete probability - that any accceptable configuration could occur on any opportunity we have the following:

In general we have a total of $N_{Oi} = \sum_{j=1}^{m} n_{Oij}$ opportunities. We wish to consider all possibilities that the particular configuration of scheduled contacts could occur on any of the opportunities over the network. It can be shown that this is achieved by using the negative binomial distribution when all the conflict-free probabilities are the same. In our case, we need to accommodate the more general case of varying conflict-free probabilities. We do this by using the above equation, but varying the upper index, n_{Ri}, on the sums from n_{Ri} to $N_{Oi} = \sum_{j=1}^{m} n_{Oij}$. (It can be demonstrated that in the case of equal probabilities across the network, this is equivalent to the cumulative probability of the negative binomial distribution with the appropriate terms; see, for example, Freund, *Mathematical Statistics*, p. 180.) Thus, the probability that satellite i may have all its contacts scheduled across the network is the sum

$$P_i\left(n_{Ri}, N_{Oi}, m, p_{ij}^{\bar{x}}\right) = \sum_{k=n_{Ri}}^{N_{Oi}} \underline{p}_i\left(k, n_{Oij}, m, p_{ij}^{\bar{x}}\right).$$

The proof of the above relationships is tedious; it results from the expansion of the following factors:

$$(p_{i1}+q_{i1})^{n_{Oi1}}(p_{i2}+q_{i2})^{n_{Oi2}}(p_{i3}+q_{i3})^{n_{Oi3}}\cdots(p_{im}+q_{im})^{n_{Oim}}$$

Each of the m parenthesized factors in the above product can be expanded using the binomial theorem with the result that each term in the particular expansion has the binomial coefficient, resulting in a polynomial of degree n_{Oij}. When all the polynomials are multiplied together per the equation, there results a single polynomial consisting of a sequence of products of m binomial coefficients multiplied by the corresponding probability factors $p_{ij}^{x_j} q_{ij}^{n_{Oij}-x_j}$, $j=1,2,\ldots m$. The sums of all these product terms correspond to a general form of the expression above, and when the constraints are applied, the result is the equation for $p_i\left(n_{Ri}, n_{Oij}, m, p_{ij}^{\overline{x}}\right)$. We refer to this as a generalized or *heterogeneous binomial distribution*[9]. Note that if the probabilities are all equal (i.e., $p_{ij}\equiv p$), the above product reduces to

$$(p_{i1}+q_{i1})^{n_{Oi1}}(p_{i2}+q_{i2})^{n_{Oi2}}(p_{i3}+q_{i3})^{n_{Oi3}}\cdots(p_{im}+q_{im})^{n_{Oim}}=(p+q)^{N_{Oi}},\ N_{Oi}=\sum_{j=1}^{m} n_{Oij},$$

which, on expansion, yields the standard binomial distribution.

Expected Values and Standard Deviation.

A useful item of information is the expected number of contacts that could be scheduled conflict-free on the network of m stations. In this case of discrete probabilities, a general form for the expected value of an event x with individual probability of occurrence $p(x)$ is the well-known expression

$$E(x)=\sum_{x=1}^{n} x\,p(x).$$

In the case of the standard binomial distribution the mean or expected value is

$$\mu=E(x)=\sum_{x=1}^{n} x\binom{n}{x}p^x(1-p)^{n-x}=np.$$

In the case of our heterogeneous binomial distribution we use the same approach, where here we use n_{Ci}, the number of schedulable contacts for satellite i, as the random variable x. It can also be shown that the expected number of conflict-free schedulable contacts for satellite i is just

$$\hat{n}_{Ci}=E(n_{Ci})=n_{Ri}\,P_i\left(n_{Ri}, N_{Oi}, m, p_{ij}^{\overline{x}}\right),$$

which conforms nicely to the form for the standard binomial distribution.

For the variance of the binomial distribution, the well-known result

[9] A search of the literature has yet to reveal if the distribution developed here has been derived by elsewhere. Absent that possibility, we have chosen to refer to the distribution used here as "heterogeneous" to indicate the fact that the distribution is similar to the standard binomial distribution, but contains a mixture of probabilities. In this sense it is a generalization of the standard binomial distribution; it reduces to the standard binomial when all the probabilities are equal.

$$E\left[(x-\mu)^2\right]=\sum_{x=1}^{n}(x-\mu)^2\binom{n}{x}p^x(1-p)^{n-x}=np(1-p)=npq$$

becomes, for satellite *i*,

$$v_{n_{Ci}}=E\left[(x-n_{Ci})^2\right]=n_{Ri}P_i\left(n_{Ri},N_{Oi},m,p_{ij}^{\bar{x}}\right)\left[1-P_i\left(n_{Ri},N_{Oi},m,p_{ij}^{\bar{x}}\right)\right]$$

in the case of our heterogeneous binomial distribution.

To determine the expected values and variances for the complete suite of *n* satellites we use the same standard definitions for mean and variance, resulting in the

Mean: $$\hat{N}_C=\sum_{i=1}^{n}n_{Ri}P_i\left(n_{Ri},N_{Oi},m,p_{ij}^{\bar{x}}\right)=\sum_{i=1}^{n}\hat{n}_{Ci}$$

Variance: $$V_{N_C}=\sum_{i=1}^{n}\left(n_{Ci}-\hat{n}_C\right)^2P_i\left(n_{Ri},N_{Oi},m,p_{ij}^{\bar{x}}\right)$$

Putting It All Together.

Given:

- Network of *m* tracking stations at latitudes ϕ_j.
- Suite of *n* satellites in circular orbits at altitudes h_i, orbit inclinations i_i, and requiring n_{Ri} daily contacts

Calculate:

- Satellite FOVs, $\frac{w_0}{2}$, for each satellite-station combination.
- Expected value of the arc distance traveled, $\hat{\alpha}$, during a station pass for each satellite-station combination.
- Probability, $\frac{\Delta\Lambda}{\pi}$, of a station pass based on the geometry of the satellite orbit RAAN lying within the longitude bands, for each satellite-station combination.
- Probability of a satellite being within the station FOV, for each satellite-station combination.
- Probability, $p_{ci}=\frac{n_{Ri}}{N_{Oi}}=\frac{n_{Ri}}{\sum_{j=1}^{m}n_{Oij}}$, of a satellite needing/requesting a contact, for each satellite.
- Combine these three probabilities to obtain the conflict-free probabilities $p_{ij}^{\bar{x}}$, and failures, or conflicts, $p_{ij}^{x}=q_{ij}^{\bar{x}}=\left(1-p_{ij}^{\bar{x}}\right)$, for each satellite-station combination.
- Using these probabilities, compute the individual probabilities of each satellite being scheduled across the network of *j* stations, and there cumulative probabilities of success over the range n_{Ri} to N_{Oij}, keeping in

mind the constraints on the values of the x_j: $\sum_{j=1}^{m} x_j = n_{Ri}$ and $x_j \le n_{Oij}$, and remembering that

$$\binom{n}{k} \equiv 0, k > n.$$

$$\underline{p}_{si}\left(n_{Ri}, n_{Oij}, m, p_{ij}^{\bar{x}}\right) = \sum_{x_1=0}^{n_{ri}} \sum_{x_2=0}^{n_{Ri}-x_1} \sum_{x_3=0}^{n_{Ri}-x_1-x_2} \cdots \sum_{x_{m-1}=0}^{n_{Ri}-x_1-x_2-\ldots-x_{m-2}} \prod_{j=1}^{m} \binom{n_{Oij}}{x_j} p_{sij}^{x_j} q_{sij}^{n_{Oij}-x_j} \; ; \; q_{sij} \square 1 - p_{sij}$$

$$P_{si}\left(n_{Ri}, N_{Oi}, m, p_{ij}^{\bar{x}}\right) = \sum_{k=n_{Ri}}^{N_{Oi}} \underline{p}_{si}\left(k, n_{Oij}, m, p_{ij}^{\bar{x}}\right)$$

- Finally, using the P_i above, compute the total probability of being able to schedule *all* satellites conflict-free across the network: $P_{AllSats} = \prod_{i=1}^{n} P_{si}$.

Appendix B

Monte Carlo MATLAB Code

The following is the exact code from the MATLAB program written.

```
%Matthew Stubbe
%Satellite Thesis
%
%Main Program

clear;
clc;
tic;

ITERATIONS=25;




%Givens
MU = 3.986*10^5;
RE = 6378;
WE = 7.292123517 * 10^-5;
TOTREQCON= 0;

%User Inputs
SATN=  10;%abs(input('Enter the number of Satellites: '));

SATH= xlsread('10sats.xls');

SATI= xlsread('10incs.xls');

for i=1:SATN

 %   SATH(i,1)=  input(sprintf('Enter the height of orbit(km) of station # %g: ', i));

 %   SATI(i,1)=  input(sprintf('Enter the inclination of orbit(DEG) of station # %g: ', i));

   SATI(i,1)= SATI(i,1)/180*pi;

   SATTT(i,1)=360;% input(sprintf('Enter the minimum tracking time(sec.) of station #
%g: ', i));

   REQCON(i,1)=18;% input(sprintf('Enter the number of required contacts per day of
station # %g: ', i));
```

```
end

clc;

STNN= abs(input('Enter the number of Stations: '));

for i=1:STNN

   STNL(i,1)=  input(sprintf('Enter the latitude(DEG) of station # %g: ', i));

   STNL(i,1)= STNL(i,1)/180*pi;

   STNLO(i,1)= input(sprintf('Enter the longitude(DEG) of station # %g: ', i));

   STNLO(i,1)= STNLO(i,1)/180*pi;

end

clc;

DURATION=72;%abs(input('Enter the duration of simulated run time(hrs): '));

TSS=15;%abs(input('Enter the time step size(sec): '));

TS=DURATION*3600/TSS;

%Random Variables

for q=1:ITERATIONS

for i=1:SATN

   OMEGA(i,1)=rand(1)*2*pi;

   U0(i,1)=rand(1)*2*pi;

   r(i,1)= SATH(i,1) + RE;

end

%Beginning Calculations

COUNT= 0;

TOTREQCON= sum(REQCON);
```

```
%R

R=RE;

%r
for i=1:SATN

   THETADOT(i,1)= sqrt(MU/(r(i,1)^3));

   for j=1:TS

      COUNT= j;

      THETA(i,j)= U0(i,1) + (THETADOT(i,1) * TSS * COUNT);

   end
end

for j=1:TS

   COUNT= j;

   for i=1:STNN

      LONG(i,j)= STNLO(i,1) + WE*TSS*COUNT;

   end
end

%Satellite track time which gives WNOT/2(WNOTF)

for i=1:SATN

   SATTRACK(i,1)= SATTT(i,1) * THETADOT(i,1);

   ALPHA(i,1)= acos(R/r(i,1));

   WNOTF(i,1)=acos((cos(ALPHA(i,1)))/(cos(SATTRACK(i,1)/2)));

end

%rbar and RBAR
for k=1:TS

   for j=1:STNN
```

```
      RBAR(1,1)= RE*(cos(STNL(j,1))*cos(LONG(j,k)));

      RBAR(1,2)= RE*(cos(STNL(j,1))*sin(LONG(j,k)));

      RBAR(1,3)= RE*(sin(STNL(j,1)));

        for i=1:SATN

            rbar(1,1)= r(i,1)*((cos(OMEGA(i,1))*cos(THETA(i,k)))
(sin(OMEGA(i,1))*sin(THETA(i,k))*cos(SATI(i,1))));

            rbar(1,2)= r(i,1)*((sin(OMEGA(i,1))*cos(THETA(i,k))) +
(cos(OMEGA(i,1))*sin(THETA(i,k))*cos(SATI(i,1))));

            rbar(1,3)= r(i,1)*(sin(THETA(i,k))*sin(SATI(i,1)));


            Q = dot(RBAR,rbar);

            SI(i,j,k)=acos(Q/(r(i,1)*R));

      end
    end
end

%Determining Contacts and Contact Times

CONTACT= zeros(SATN,STNN);
CONTACTLENGTH= zeros(SATN,STNN);
CONTACTSTART= zeros(SATN,STNN);
CONTACTEND= zeros(SATN,STNN);
ZERK= zeros(SATN,STNN);

for i=1:SATN
    for j=1:STNN

      KEEP=0;

      COUNTER=0;

      if CONTACT(i,j)-floor(CONTACT(i,j))== 0.5

          CONTACT(i,j)=floor(CONTACT(i,j));
      end

      for k=2:TS
```

```
ZERK(k,1)= ((SI(i,j,k)-WNOTF(i,1))*(SI(i,j,k-1)-WNOTF(i,1)));

if SI(i,j,k) < ALPHA(i,1) & ZERK(k,1) > 0

   COUNTER=COUNTER+1;

elseif ZERK(k,1) < 0

   CONTACT(i,j)= CONTACT(i,j) + 0.5;

   COUNTER=COUNTER+1;

   KEEP=KEEP+1;

elseif SI(i,j,k)> ALPHA(i,1) & KEEP > 1

   if CONTACT(i,j)-floor(CONTACT(i,j))== 0.5

      CONTACT(i,j)=floor(CONTACT(i,j));
   end


   CONTACTLENGTH(i,j,floor(CONTACT(i,j)))= (COUNTER-1)*TSS;

   CONTACTSTART(i,j,floor(CONTACT(i,j)))=  (k - COUNTER)*TSS;

   CONTACTEND(i,j,floor(CONTACT(i,j)))=  (k - 1)*TSS;

   KEEP = 0;

   COUNTER = 0;

elseif SI(i,j,k)> ALPHA(i,1) & KEEP <= 1

   if CONTACT(i,j)-floor(CONTACT(i,j))== 0.5

      CONTACT(i,j)=floor(CONTACT(i,j));
   end

   COUNTER = 0;

   KEEP = 0;


end
```

```
    end
  end
end

COUNTP(q,1)=0;

for j=1:STNN

  for i=1:SATN

    COUNT2= COUNTP(q,1);

    for k=1:CONTACT(i,j)

      COUNTP(q,1)= k + COUNT2;

      VISIBILITIES(COUNTP(q,1),1)= i;

      VISIBILITIES(COUNTP(q,1),2)= j;

      VISIBILITIES(COUNTP(q,1),3)= CONTACTSTART(i,j,k);

      VISIBILITIES(COUNTP(q,1),4)= CONTACTEND(i,j,k);

      VISIBILITIES(COUNTP(q,1),5)= CONTACTLENGTH(i,j,k);

    end
  end
end

%Sort organized data by end times

FINAL= sortrows(VISIBILITIES, 4);


%Schedule contacts

SCHEDULEDCONTACTS= zeros(SATN,1);
X= zeros(STNN,1);
P= zeros(STNN,1);
PLACEMENT= 0;

for i= 1:COUNTP(q,1)

  for j= 1:STNN
```

```
      if SCHEDULEDCONTACTS(FINAL(i,1),1)< REQCON(FINAL(i,1),1)

        if j== FINAL(i,2)

          if X(j,1)<= FINAL(i,3)

            PLACEMENT= PLACEMENT + 1;

            P(j,1)= P(j,1)+1;

            for k= 1:5

              SCHEDULE(PLACEMENT,k)= FINAL(i,k);

            end

            X(j,1)= FINAL(i,4);

            SCHEDULEDCONTACTS(FINAL(i,1),1)=
SCHEDULEDCONTACTS(FINAL(i,1),1)+1;

          end
        end
      end
   end
end


%Performance Ratios

TOTSCHCON(q,1)= sum(SCHEDULEDCONTACTS);

for i= 1:SATN
   ASC(i,q)= SCHEDULEDCONTACTS(i,1);
end

%ODCR(q,1)= COUNTP(q,1)/TOTREQCON;
%SPR(q,1)= TOTSCHCON(q,1)/COUNTP(q,1);
%SDCR(q,1)= TOTSCHCON(q,1)/TOTREQCON;
%SLACR= TOTSCHCON/COUNT;

%clc;

%fprintf('\n\nThe overall "Opportunity/Demand Count Ratio" is %3.1f/%3.1f or
%3.6f\n', COUNT, TOTREQCON, ODCR);
```

```
%fprintf('The overall "Scheduler Performance Ratio" is %3.1f/%3.1f or %3.6f\n',
TOTSCHCON, COUNT, SPR);

%fprintf('The overall"Schedule/Demand Count Ratio" is %3.1f/%3.1f or %3.6f\n\n',
TOTSCHCON, TOTREQCON, SDCR);

%fprintf('The "Site Loading/Availability Count Ratio" is %3.1f/%3.1f or %3.6f',
TOTSCHCON, COUNT, SLACR);


%disp('The optimal schedule is...');


%disp('        Sat       Site   Start Time   Stop Time    Duration ');

%disp(SCHEDULE);




for i=1:SATN

   %fprintf('The number of scheduled contacts/required contacts for Satellite %g is
%3.1f/%3.1f or %3.6f\n', i, SCHEDULEDCONTACTS(i,1), REQCON(i,1),
SCHEDULEDCONTACTS(i,1)/REQCON(i,1));

   OPPORTUNITIES(i,q)=0;

   for j=1:STNN

      OPPORTUNITIES(i,q)= floor(CONTACT(i,j))+OPPORTUNITIES(i,q);

   %fprintf('The number of scheduled contacts/opportunities for Satellite %g is
%3.1f/%3.1f or %3.6f\n', i, SCHEDULEDCONTACTS(i,1), OPPORTUNITIES,
SCHEDULEDCONTACTS(i,1)/OPPORTUNITIES);

   end

end




end
```

```
AVECOUNT=sum(COUNTP)/ITERATIONS;

AVETOTSCHCON=sum(TOTSCHCON)/ITERATIONS;

fprintf('\n\nThe average overall "Feasibility Ratio" is %3.1f/%3.1f or %3.6f\n',
AVECOUNT, TOTREQCON, AVECOUNT/TOTREQCON);

STDDEVALLOPP= std(COUNTP,0,1);

fprintf('The standard deviation of opportunities for all Satellites is %3.1f\n',
STDDEVALLOPP);

fprintf('The average overall "Utilization Ratio" is %3.1f/%3.1f or %3.6f\n',
AVETOTSCHCON, AVECOUNT, AVETOTSCHCON/AVECOUNT);

fprintf('The average overall "Performance Ratio" is %3.1f/%3.1f or %3.6f\n',
AVETOTSCHCON, TOTREQCON, AVETOTSCHCON/TOTREQCON);

STDDEVALLSCHCON= std(TOTSCHCON,0,1);

fprintf('The standard deviation of scheduled contacts for all Satellites is %3.1f\n\n',
STDDEVALLSCHCON);


AVEOPPORTUNITIES= mean(OPPORTUNITIES,2);
STDDEVSCHCON = std(ASC,0,2);
STDDEVOPP = std(OPPORTUNITIES,0,2);

for i=1:SATN

   ASC2=0;

for v=1:q
   ASC2=ASC(i,v)+ASC2;
end

   AVESCHEDULEDCONTACTS(i,1)= ASC2/ITERATIONS;



   fprintf('The average number of scheduled contacts/required contacts for Satellite %g is
%3.1f/%3.1f or %3.6f\n', i, AVESCHEDULEDCONTACTS(i,1), REQCON(i,1),
AVESCHEDULEDCONTACTS(i,1)/REQCON(i,1));
```

```
   fprintf('The standard deviation of scheduled contacts for Satellite %g is %3.1f\n', i,
STDDEVSCHCON(i,1));

   fprintf('The average number of scheduled contacts/opportunities for Satellite %g is
%3.1f/%3.1f or %3.6f\n', i, AVESCHEDULEDCONTACTS(i,1),
AVEOPPORTUNITIES(i,1),
AVESCHEDULEDCONTACTS(i,1)/AVEOPPORTUNITIES(i,1));



   fprintf('The standard deviation of opportunities for Satellite %g is %3.1f\n', i,
STDDEVOPP(i,1));
end

toc;
```

Appendix C

Orbital Mechanics

In order to verify the probabilistic approach, calculations of actual satellite/station visibilities are needed. Because of the fact the satellites are orbiting earth a few values can be taken as known in these calculations. These values can be found in any basic orbital mechanics book and are as follows:

$$\mu_E = \text{Gravitational Constant of Earth} \left[\frac{\text{km}^3}{\text{sec}^2}\right]$$
$$R_e = \text{Radius of Earth} \left[\text{km}\right]$$
$$\omega_E = \text{Rotational Velocity of Earth} \left[\frac{\text{rad}}{\text{sec}}\right]$$

Next in order to continue some information about the satellites and stations being tested is needed. The needed values are:

$$h_{sat} = \text{Height of Sat Orbit} \left[\text{km}\right]$$
$$i_{sat} = \text{Inclination of Sat Orbit} \left[\text{rad}\right]$$
$$t_{sat} = \text{Minimum Required Tracking Time for Viable Contact} \left[\text{sec}\right]$$
$$\phi_{sat} = \text{Latitude of Station} \left[\text{rad}\right]$$
$$\lambda_{sat} = \text{Longitude of Station} \left[\text{rad}\right]$$
$$\tau = \text{Time Elapsed at Evaluation} \left[\text{sec}\right]$$

Also for this simulation two variables were randomly generated for every run of the calculations. This randomization adds the uncertainty of launch dates by randomizing the parameters which are directly correlated with that date.

$$\Omega = \text{Right Ascension of the Ascending Node} \left[\text{rad}\right]$$
$$U_0 = \text{Initial Argument of Latitude for the Satellite} \left[\text{rad}\right]$$

Now that all the necessary initial values have been identified the calculations can begin. These calculations will begin by finding the radius of the station and satellite from the center of the earth. Since the station altitudes were ignored the radius of the station is identical to the radius of the earth as shown below.

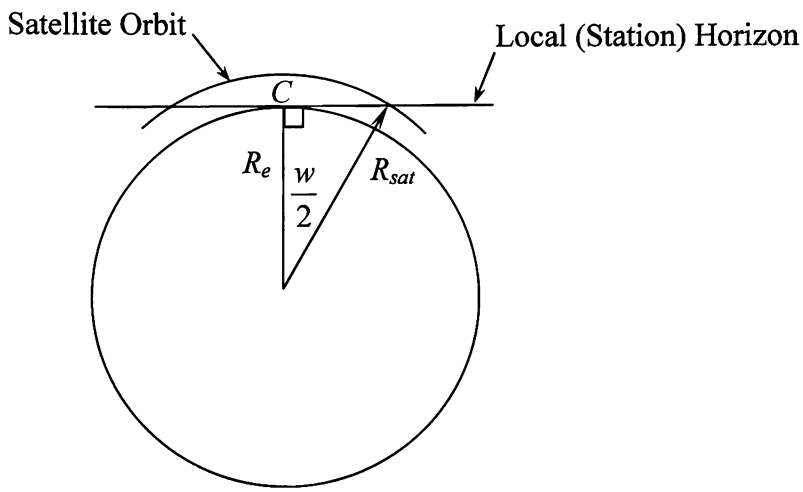

Figure 26 - Radii

$$
\begin{aligned}
R_{stn} &= \text{Radius of Station } [\text{km}] \\
R_{stn} &= R_e \\
R_{sat} &= \text{Radius of Satellite } [\text{km}] \\
R_{sat} &= R_e + h_{sat}
\end{aligned}
$$

Once the radii are know the rotational velocity for the satellite can be found.

$$
\begin{aligned}
\dot{\theta}_{sat} &= \text{Rotational Velocity of Satellite } \left[\frac{\text{rad}}{\text{sec}}\right] \\
\dot{\theta}_{sat} &= \left(\frac{\mu_{\mathrm{E}}}{R_e^3}\right)^{1/2}
\end{aligned}
$$

Once the rotational velocity is found the amount of distance the satellite travels during the minimum tracking time, in terms of radians, can be determined as follows:

$$\theta_{track} = \text{Tracking Distance } [\text{rad}]$$
$$\theta_{track} = \dot{\theta}_{sat}\ t_{sat}$$

Also the position of the satellite in its orbit at the given time can be found using the rotational velocity.

$$\theta_{sat\ postion} = \theta_{sp} = \text{Angle of Satellite in Orbit from Equator } [\text{rad}]$$
$$\theta_{sp} = U_0 + \dot{\theta}_{sat}\tau$$

Now the longitude of the ground station at the given time is also calculated.

$$\lambda_{sat\ position} = \lambda_{sp} = \text{Angle of Station from 0 degree line } [\text{rad}]$$
$$\lambda_{sp} = \lambda_{stn} + \omega_E \tau$$

Next the "cone angle" (FOV), which is the maximum amount of distance a satellite will be in view of a ground station, is determined. Note that all stations would have the same "cone angle" with a particular satellite since all stations are considered to have the same radius.

$$\frac{w}{2} = \text{Satellite Visiblity "Cone Angle"}$$
$$\frac{w}{2} = \cos\left(\frac{R_{stn}}{R_{sat}}\right)$$

The previous calculation gave the maximum angle for each satellite but every time a satellite passes within view of a station it doesn't occur with the maximum angle possible. Since there is a minimum requirement for the amount of time a satellite must be over a station in order to make a valid contact, which was converted to the angle θ_{track},

it is now necessary to determine the minimum "cone angle" for a valid contact.

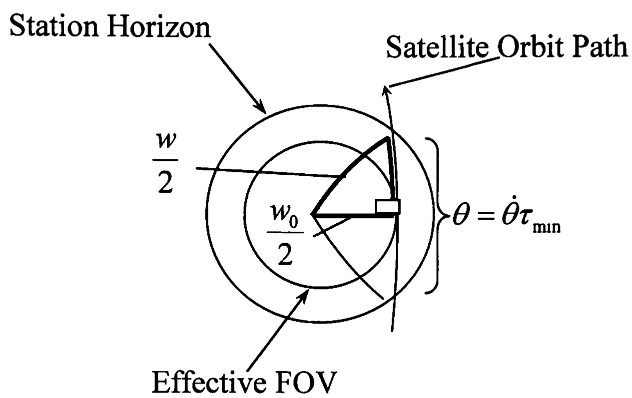

Figure 27 - Effective FOV

$$\frac{w_0}{2} = \text{Satellite Minimum "Cone Angle" for a Contact } [\text{rad}]$$

$$\frac{w_0}{2} = \cos^{-1}\left(\frac{\cos\left(\frac{w}{2}\right)}{\cos\left(\frac{\theta_{track}}{2}\right)}\right)$$

The next step is to determine the vector locations of each satellite and station so the angle between them can be calculated and compared to $\frac{w_0}{2}$.

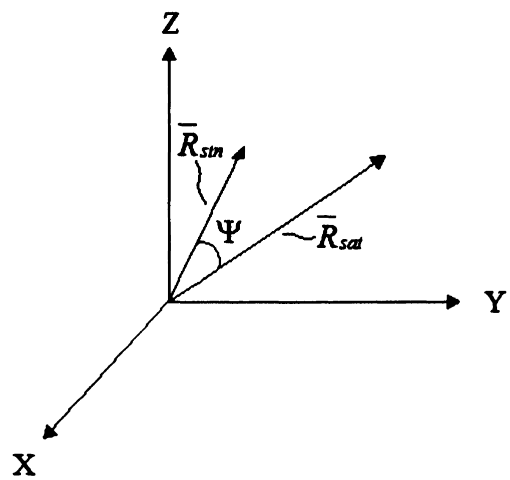

Figure 28 - Angle Between Station and Satellite

$$\overline{R}_{sat} = \text{Vector Location of Satellite}$$

$$\overline{R}_{sat} = \begin{Bmatrix} \cos\phi_{stn}\cos\lambda_{sp} \\ \cos\phi_{stn}\sin\lambda_{sp} \\ \sin\phi_{stn} \end{Bmatrix} \{X\ Y\ Z\}$$

$$\overline{R}_{stn} = \text{Vector Location of Station}$$

$$\overline{R}_{stn} = \begin{Bmatrix} \cos\Omega\cos\theta_{sp} - \sin\Omega\sin\theta_{sp}\cos i_{sat} \\ \sin\Omega\cos\theta_{sp} - \cos\Omega\sin\theta_{sp}\cos i_{sat} \\ \sin\theta_{sp}\sin i_{sat} \end{Bmatrix} \{X\ Y\ Z\}$$

$$\Psi = \text{Angle between Satellite and Station}$$

$$\Psi = \cos^{-1}\left(\frac{\overline{R}_{sat} \cdot \overline{R}_{stn}}{R_{sat}R_{stn}}\right)$$

Comparing Ψ and $\frac{w_0}{2}$ will determine whether a viable contact has occurred. If $\Psi \le \frac{w_0}{2}$ a usable contact has occurred but if $\Psi > \frac{w_0}{2}$ no contact has occurred. This value needs to be checked at every time step to determine if a contact has occurred. This is the extent of the orbital mechanics involved. The only other computations are compiling the usable contacts and applying the Greedy Selector algorithm to schedule these contacts.

Appendix D

The Greedy Activity Selector (GAS)

Greedy algorithms don't always produce optimal solutions. However, the Greedy Activity Selector always finds an optimal solution to an instance of the activity selection problem.

The following is a reproduction of a proof by Burrowbridge [4].

Let S={1,2,...,n} be the set of activities to schedule. Since we are assuming that the activities are in order by finish time, activity 1 has the earliest finish time. We wish to show that there is an optimal solution that begins with a greedy choice, that is, with activity 1.

Suppose that A is an optimal solution to the problem S the given instance of the activity-selection problem, and let us order the activities in A by finish time. Suppose further that the first activity in A is activity k. If k=1, then schedule A begins with a greedy choice. If $k \neq 1$, we want to show that there is another optimal solution B to S that begins with the greedy choice, activity 1. Let B= A-{k}∪{1}. Because $f_1 \leq f_k$, the activities in B are disjoint, and since B has the same number of activities as A, it is also optimal. Thus, B is an optimal solution for S that contains the greedy choice of activity 1. Therefore, we have shown that there always exists an optimal schedule that begins with a greedy choice. Moreover, once the greedy choice of activity 1 is made, the problem reduces to finding an optimal solution for the activity-selection problem over those activities in S that are compatible with activity 1. That is, if A is an optimal solution to the original problem S, then A' = A-{1} is an optimal solution to the activity-selection problem $S' = \{i \in S: s_i \geq f_1\}$. Why? If we could find a solution B' to S' with more activities than A', adding activity 1 to B' would yield a solution B to S with more activities than A, thereby contradicting the optimality of A. Therefore, after each greedy choice is made, we are left with an optimization problem of the same form as the original problem. By induction on the number of choices made, making the greedy choice at every step produces an optimal solution.

Appendix E

MATLAB Results Example

First 10 Sats
First 2 Global Stns

The average overall "Feasibility Ratio" is 341.8/180.0 or 1.898667
The standard deviation of opportunities for all Satellites is 3.4
The average overall "Utilization Ratio" is 160.7/341.8 or 0.470154
The average overall "Performance Ratio" is 160.7/180.0 or 0.892667
The standard deviation of scheduled contacts for all Satellites is 3.3

The average number of scheduled contacts/required contacts for Satellite 1 is 18.0/18.0 or 1.000000
The standard deviation of scheduled contacts for Satellite 1 is 0.0
The average number of scheduled contacts/opportunities for Satellite 1 is 18.0/46.3 or 0.388601
The standard deviation of opportunities for Satellite 1 is 2.0
The average number of scheduled contacts/required contacts for Satellite 2 is 5.4/18.0 or 0.297778
The standard deviation of scheduled contacts for Satellite 2 is 1.0
The average number of scheduled contacts/opportunities for Satellite 2 is 5.4/6.2 or 0.864516
The standard deviation of opportunities for Satellite 2 is 0.8
The average number of scheduled contacts/required contacts for Satellite 3 is 12.9/18.0 or 0.715556
The standard deviation of scheduled contacts for Satellite 3 is 2.0
The average number of scheduled contacts/opportunities for Satellite 3 is 12.9/14.8 or 0.867925
The standard deviation of opportunities for Satellite 3 is 1.1
The average number of scheduled contacts/required contacts for Satellite 4 is 17.4/18.0 or 0.964444
The standard deviation of scheduled contacts for Satellite 4 is 1.3
The average number of scheduled contacts/opportunities for Satellite 4 is 17.4/26.8 or 0.647761
The standard deviation of opportunities for Satellite 4 is 1.0
The average number of scheduled contacts/required contacts for Satellite 5 is 18.0/18.0 or 1.000000
The standard deviation of scheduled contacts for Satellite 5 is 0.0
The average number of scheduled contacts/opportunities for Satellite 5 is 18.0/38.6 or 0.466805
The standard deviation of opportunities for Satellite 5 is 1.0
The average number of scheduled contacts/required contacts for Satellite 6 is 17.8/18.0 or 0.986667
The standard deviation of scheduled contacts for Satellite 6 is 0.9
The average number of scheduled contacts/opportunities for Satellite 6 is 17.8/25.5 or 0.695925
The standard deviation of opportunities for Satellite 6 is 1.5

The average number of scheduled contacts/required contacts for Satellite 7 is 17.3/18.0 or 0.962222
The standard deviation of scheduled contacts for Satellite 7 is 1.7
The average number of scheduled contacts/opportunities for Satellite 7 is 17.3/25.5 or 0.678683
The standard deviation of opportunities for Satellite 7 is 1.4
The average number of scheduled contacts/required contacts for Satellite 8 is 18.0/18.0 or 1.000000
The standard deviation of scheduled contacts for Satellite 8 is 0.0
The average number of scheduled contacts/opportunities for Satellite 8 is 18.0/28.8 or 0.624133
The standard deviation of opportunities for Satellite 8 is 1.2
The average number of scheduled contacts/required contacts for Satellite 9 is 18.0/18.0 or 1.000000
The standard deviation of scheduled contacts for Satellite 9 is 0.0
The average number of scheduled contacts/opportunities for Satellite 9 is 18.0/63.6 or 0.283197
The standard deviation of opportunities for Satellite 9 is 1.3
The average number of scheduled contacts/required contacts for Satellite 10 is 18.0/18.0 or 1.000000
The standard deviation of scheduled contacts for Satellite 10 is 0.0
The average number of scheduled contacts/opportunities for Satellite 10 is 18.0/65.6 or 0.274390
The standard deviation of opportunities for Satellite 10 is 0.9
Elapsed time is 491.908000 seconds.
>>

Appendix F

Probabilistic Results Example

Hagar Method 10-2-6

Input Data:..
sats= 10 , # stns= 2 , min trk time(secs)= 360
End Input..
***Sat (2) requires 6 contacts; only 2.203069 opportunities!
***Sat (3) requires 6 contacts; only 4.8059 opportunities!
Setting # of requests to # of opportunities for sat. 2 = 2
Setting # of requests to # of opportunities for sat. 3 = 5

Summary:..
Estimate of total probability that all sats are schedulable over network: 0
(This is the probability that all the reqd contacts for all satellites can be supported, conflict-free, by one or more stations in the network.

Expected total number of conflict-free contacts schedulable: 52.1248979568481
(Out of a total of 60 required contacts.)
Std dev in total conflict-free contacts for all sats: 46.1248979568481

Expected number of total opportunities for all satellites: 120.035587310791
Network scheduling efficiency (1-contacts/opportunities): 56.5754630567278 %

Expected values of performance measures:
*Expected Feasibility Ratio: 2.00059312184652
*Expected Utilization Ratio: 0.434245369432722
*Expected Performance Ratio: 0.868748299280802

Probability all contacts of at least one sat can have all contacts scheduled across network: 1
(This is the probability that the reqd contacts for at least one satellite can be supported, conflict-free, by one or more stations in the network.
Total probability of all required satellite contacts being scheduled conflict-free: 0
....
Execution time (min): 1.041667E-03

Appendix G

Additional Results

The following are the remaining results tables for the cases not contained in the Results section.

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	114.1	158.1	219.0	260.9
20	252.0	327.6	468.1	535.3
30	372.5	485.1	692.9	792.9
40	517.1	654.3	939.2	1057.0
50	617.9	793.3	1135.0	1283.3
60	751.5	962.5	1376.7	1554.0

Table 16 - Opportunities, Global Stations, Monte Carlo, 10/Day

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	104.5	144.1	198.5	254.1
20	204.8	271.6	403.8	511.3
30	299.8	404.6	574.1	752.8
40	386.6	511.1	747.7	1000.3
50	475.9	632.7	911.1	1211.4
60	576.5	764.9	1105.3	1467.7

Table 17 - Opportunities, Western Hemisphere Stations, Monte Carlo, 6/Day

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	104.4	144.0	198.6	253.9
20	204.8	270.7	386.9	511.8
30	298.9	403.8	573.5	753.1
40	387.0	511.0	748.1	1000.0
50	475.9	633.1	912.6	1210.8
60	576.0	764.6	1104.6	1467.2

Table 18 - Opportunities, Western Hemisphere Stations, Monte Carlo, 10/Day

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	120.04	165.47	230.54	272.86
20	267.18	345.51	494.59	563.84
30	395.16	512.55	732.76	835.57
40	548.81	693.20	996.32	1115.78
50	655.28	838.50	1201.02	1352.83
60	797.43	1016.55	1457.99	1639.92

Table 19 - Opportunities, Global Stations, Probabilistic, 10/Day

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	108.50	149.02	206.22	264.73
20	214.53	283.49	405.64	538.32
30	313.89	421.84	601.39	792.09
40	407.62	534.89	786.32	1053.66
50	500.28	661.93	957.39	1274.20
60	605.52	800.54	1160.42	1546.60

Table 20 - Opportunities, Western Hemisphere Stations, Probabilistic, 6/Day

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	108.50	149.02	206.22	264.73
20	214.53	283.49	405.64	538.32
30	313.89	421.84	601.39	792.09
40	407.62	534.89	786.32	1053.66
50	500.28	661.93	957.39	1274.20
60	605.52	800.54	1160.42	1546.60

Table 21 - Opportunities, Western Hemisphere Stations, Probabilistic, 10/Day

# of Satellites	# of Scheduled Contacts (10 Contacts/Day) # of Stations			
	2	3	4	5
10	69.8	89.2	92.8	96.1
20	121.6	165.7	190.0	194.9
30	147.3	208.3	264.9	286.2
40	168.4	236.5	316.2	360.7
50	183.0	259.1	349.7	411.5
60	195.7	277.3	377.1	452.5

Table 22 - Scheduled Contacts, Global Stations, Monte Carlo, 10/Day

# of Satellites	# of Scheduled Contacts (6 Contacts/Day) # of Stations			
	2	3	4	5
10	55.1	58.8	59.8	60.0
20	97.2	110.5	119.6	120.0
30	124.9	147.8	175.3	178.5
40	139.3	173.5	226.0	237.8
50	156.2	204.3	269.8	295.6
60	172.3	232.0	308.9	352.5

Table 23 - Scheduled Contacts, Western Hemisphere Stations, Monte Carlo, 6/Day

# of Satellites	# of Scheduled Contacts (10 Contacts/Day) # of Stations			
	2	3	4	5
10	71.1	86.6	95.7	96.9
20	108.1	140.7	182.9	194.8
30	131.1	183.7	244.3	282.6
40	143.3	205.0	282.1	351.6
50	159.2	228.8	315.6	400.6
60	173.5	251.4	346.9	439.2

Table 24 - Scheduled Contacts, Western Hemisphere Stations, Monte Carlo, 10/Day

# of Satellites	# of Scheduled Contacts (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	50.86	80.00	80.00	89.55
20	92.22	156.31	173.58	183.43
30	103.63	199.9112	259.73	269.95
40	133.91	231.56	326.46	347.08
50	142.16	239.58	366.27	408.69
60	174.82	254.16	398.86	481.82

Table 25 - Scheduled Contacts, Global Stations, Probabilistic, 10/Day

# of Satellites	# of Scheduled Contacts (6 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	52.23	56.15	56.07	59.78
20	97.41	105.20	111.77	119.39
30	129.15	151.93	166.23	174.66
40	151.53	178.52	216.02	232.54
50	178.90	210.20	258.90	286.14
60	206.75	244.05	308.56	340.86

Table 26 - Scheduled Contacts, Western Hemisphere Stations, Probabilistic, 6/Day

# of Satellites	# of Scheduled Contacts (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	54.69	77.53	93.91	93.90
20	88.40	134.42	174.39	182.29
30	116.40	178.18	236.34	267.77
40	141.66	210.46	267.90	336.70
50	151.02	236.78	298.59	389.29
60	150.30	244.84	345.74	456.48

Table 27 - Scheduled Contacts, Western Hemisphere Stations, Probabilistic, 10/Day

	Feasibility Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	1.140933	1.581067	2.189733	2.608667
20	1.260200	1.637933	2.340267	2.676667
30	1.241511	1.617156	2.309778	2.643067
40	1.292733	1.635633	2.348100	2.642533
50	1.235707	1.586560	2.270080	2.566533
60	1.252422	1.604089	2.294444	2.590044

Table 28 - Feasibility Ratio, Global Stations, Monte Carlo, 10/Day

	Feasibility Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	1.741556	2.402222	3.307778	4.235333
20	1.706778	2.263667	3.364889	4.260667
30	1.665704	2.248000	3.189630	4.182444
40	1.610722	2.129556	3.115333	4.167889
50	1.586356	2.108844	3.037111	4.038089
60	1.601481	2.124630	3.070148	4.076815

Table 29 - Feasibility Ratio, Western Hemisphere Stations, Monte Carlo, 6/Day

	Feasibility Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	1.043733	1.439600	1.986000	2.539067
20	1.024200	1.353600	1.934667	2.559067
30	0.996178	1.345911	1.911733	2.510311
40	0.967467	1.277433	1.870300	2.499900
50	0.951813	1.266293	1.825147	2.421573
60	0.959933	1.274289	1.840956	2.445356

Table 30 - Feasibility Ratio, Western Hemisphere Stations, Monte Carlo, 10/Day

# of Satellites	Feasibility Ratio (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	1.200356	1.654724	2.305390	2.728639
20	1.335924	1.727527	2.472950	2.819178
30	1.317215	1.708502	2.442517	2.785218
40	1.372037	1.732994	2.490805	2.789448
50	1.310552	1.676999	2.402043	2.705664
60	1.329048	1.694242	2.429983	2.733193

Table 31 - Feasibility Ratio, Global Stations, Probabilistic, 10/Day

# of Satellites	Feasibility Ratio (6 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	1.808292	2.483679	3.437008	4.412197
20	1.787729	2.362445	3.380320	4.486036
30	1.743817	2.343559	3.341050	4.400475
40	1.698397	2.228710	3.276347	4.390247
50	1.667601	2.206435	3.191284	4.247336
60	1.686693	2.229905	3.232367	4.308066

Table 32 - Feasibility Ratio, Western Hemisphere Stations, Probabilistic, 6/Day

# of Satellites	Feasibility Ratio (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	1.084975	1.490207	2.062205	2.647318
20	1.072637	1.417467	2.028192	2.691622
30	1.046290	1.406135	2.004630	2.640285
40	1.019038	1.337226	1.965808	2.634148
50	1.000561	1.323861	1.914770	2.548401
60	1.009205	1.334227	1.934033	2.577660

Table 33 - Feasibility Ratio, Western Hemisphere Stations, Probabilistic, 10/Day

	Utilization Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.612130	0.564092	0.423796	0.368413
20	0.482410	0.505678	0.405880	0.364010
30	0.395575	0.429368	0.382278	0.360978
40	0.325641	0.361450	0.336655	0.341263
50	0.296252	0.326577	0.308101	0.320661
60	0.260402	0.288152	0.273898	0.291200

Table 34 - Utilization Ratio, Global Stations, Monte Carlo, 10/Day

	Utilization Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.527370	0.407956	0.301377	0.236109
20	0.474383	0.406666	0.296163	0.234653
30	0.416641	0.365230	0.305248	0.237093
40	0.360466	0.339455	0.302215	0.237690
50	0.328188	0.322978	0.296129	0.244010
60	0.298867	0.303321	0.279526	0.240166

Table 35 - Utilization Ratio, Western Hemisphere Stations, Monte Carlo, 6/Day

	Utilization Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.681528	0.601741	0.481705	0.381663
20	0.527827	0.519553	0.472605	0.380660
30	0.438699	0.454942	0.425931	0.375235
40	0.370418	0.401143	0.377087	0.351654
50	0.334547	0.361328	0.345835	0.330848
60	0.301225	0.328828	0.314088	0.299361

Table 36 - Utilization Ratio, Western Hemisphere Stations, Monte Carlo, 10/Day

# of Satellites	Utilization Ratio (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	0.423692	0.483464	0.347013	0.328182
20	0.345173	0.452400	0.350967	0.325320
30	0.262257	0.390032	0.354451	0.323077
40	0.244007	0.334050	0.327661	0.311067
50	0.216949	0.285723	0.304968	0.302098
60	0.219234	0.250026	0.273566	0.293808

Table 37 - Utilization Ratio, Global Stations, Probabilistic, 10/Day

# of Satellites	Utilization Ratio (6 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	0.481407	0.376800	0.271870	0.225806
20	0.454056	0.371093	0.275545	0.221785
30	0.411440	0.360157	0.276416	0.220510
40	0.371755	0.333754	0.274726	0.220697
50	0.357608	0.317550	0.270419	0.224565
60	0.341441	0.304852	0.265901	0.220397

Table 38 - Utilization Ratio, Western Hemisphere Stations, Probabilistic, 6/Day

# of Satellites	Utilization Ratio (10 Contacts/Day)			
	# of Stations			
	2	3	4	5
10	0.504111	0.520293	0.455372	0.354696
20	0.412050	0.474150	0.429904	0.338633
30	0.370841	0.422393	0.392985	0.338054
40	0.347533	0.393466	0.340696	0.319549
50	0.301871	0.357712	0.311879	0.305518
60	0.248210	0.305844	0.297942	0.295149

Table 39 - Utilization Ratio, Western Hemisphere Stations, Probabilistic, 10/Day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.698400	0.891867	0.928000	0.961067
20	0.607933	0.828267	0.949867	0.974333
30	0.491111	0.694356	0.882978	0.954089
40	0.420967	0.591200	0.790500	0.901800
50	0.366080	0.518133	0.699413	0.822987
60	0.326133	0.462222	0.628444	0.754222

Table 40 - Performance Ratio, Global Stations, Monte Carlo, 10/Day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.918444	0.980000	0.996889	1.000000
20	0.809667	0.920556	0.996556	0.999778
30	0.694000	0.821037	0.973630	0.991630
40	0.580611	0.722889	0.941500	0.990667
50	0.520622	0.681111	0.899378	0.985333
60	0.478630	0.644444	0.858185	0.979111

Table 41 - Performance Ratio, Western Hemisphere Stations, Monte Carlo, 6/Day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.711333	0.866267	0.956667	0.969067
20	0.540600	0.703267	0.914333	0.974133
30	0.437022	0.612311	0.814267	0.941956
40	0.358367	0.512433	0.705267	0.879100
50	0.318427	0.457547	0.631200	0.801173
60	0.289156	0.419022	0.578222	0.732044

Table 42 - Performance Ratio, Western Hemisphere Stations, Monte Carlo, 10/Day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.508581	0.800000	0.800000	0.895489
20	0.461125	0.781533	0.867924	0.917136
30	0.345449	0.666371	0.865754	0.899840
40	0.334787	0.578907	0.816141	0.867704
50	0.284323	0.479157	0.732546	0.817376
60	0.291373	0.423605	0.664760	0.803035

Table 43 - Performance Ratio, Global Stations, Probabilistic, 10/Day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.870524	0.935850	0.934418	0.996300
20	0.811729	0.876687	0.931432	0.994935
30	0.717476	0.844050	0.923518	0.970348
40	0.631388	0.743841	0.900096	0.968913
50	0.596348	0.700652	0.862984	0.953803
60	0.575906	0.679791	0.859491	0.949485

Table 44 - Performance Ratio, Western Hemisphere Stations, Probabilistic, 6/Day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	0.546948	0.775345	0.939071	0.938994
20	0.441980	0.672092	0.871927	0.911472
30	0.388007	0.593941	0.787790	0.892559
40	0.354150	0.526153	0.669743	0.841739
50	0.302040	0.473561	0.597176	0.778583
60	0.250494	0.408065	0.576229	0.760793

Table 45 - Performance Ratio, Western Hemisphere Stations, Probabilistic, 10/Day

Appendix H

Percent Differences

Percent Difference Plots not contained in Analysis Section

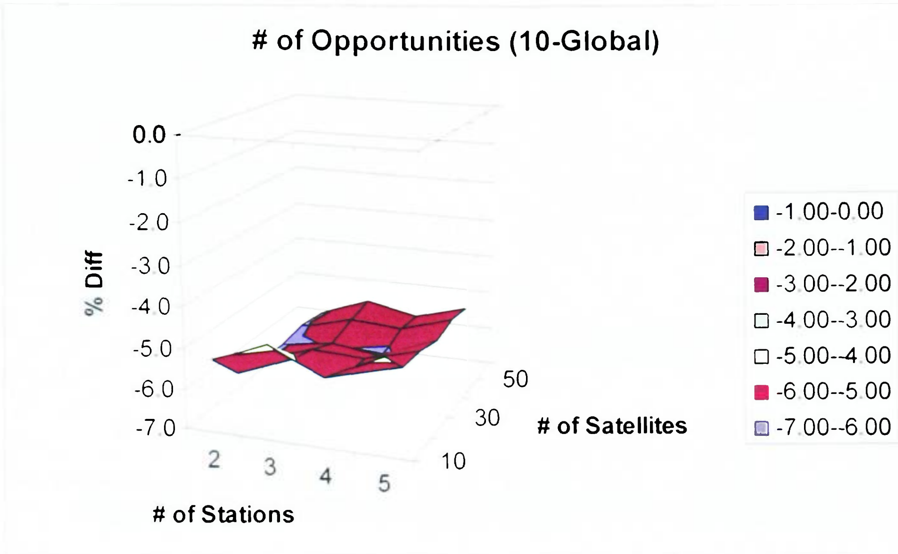

Figure 29 - Percent Difference, Opportunities, Global Stations, 10/Day

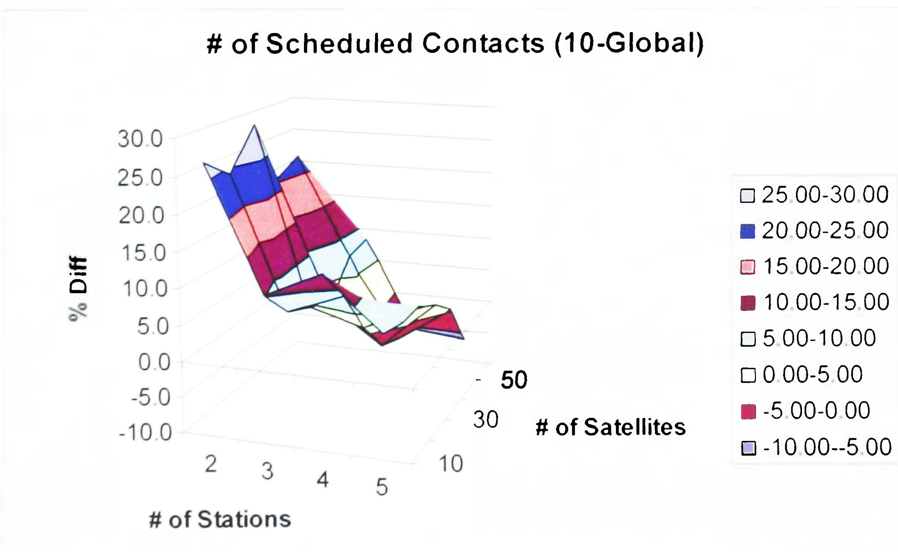

Figure 30 - Percent Difference, Scheduled Contacts, Global Stations, 10/Day

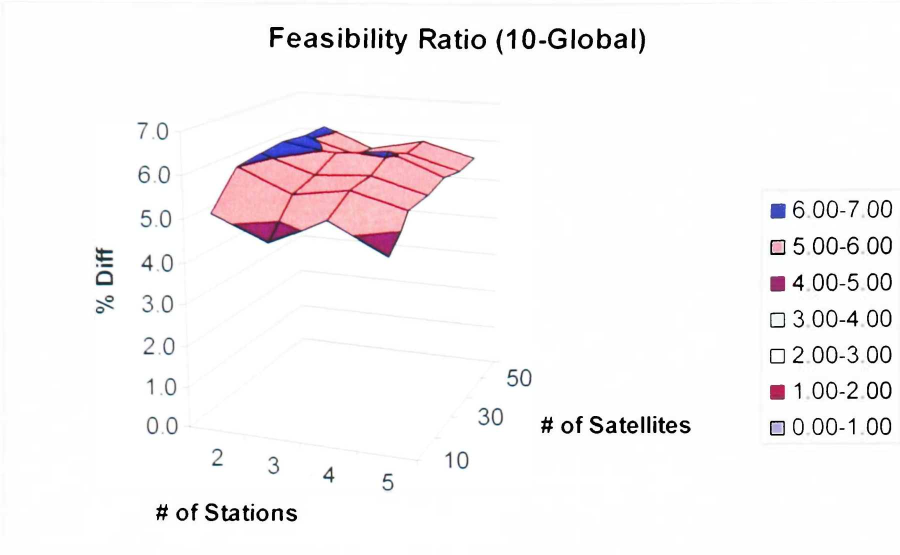

Figure 31 - Percent Difference, Feasibility, Global Stations, 10/Day

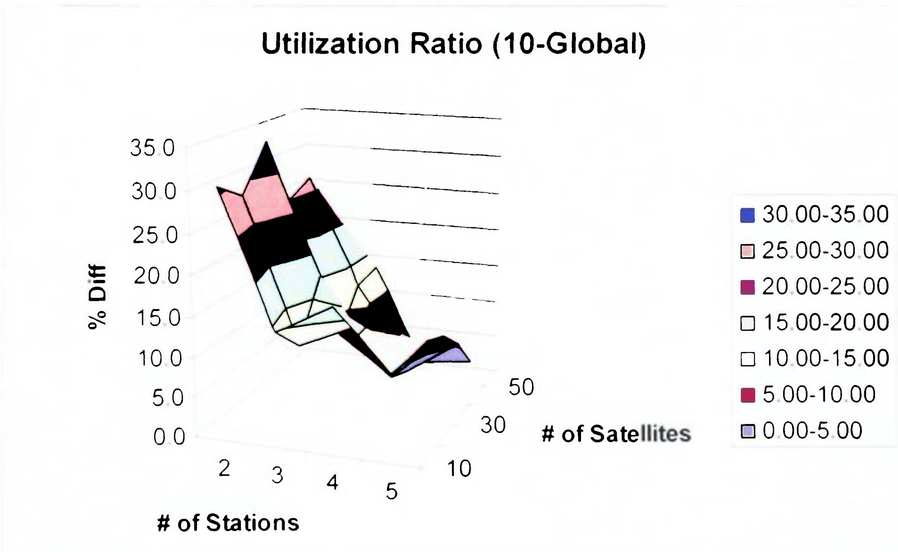

Figure 32 - Percent Difference, Utilization, Global Stations, 10/Day

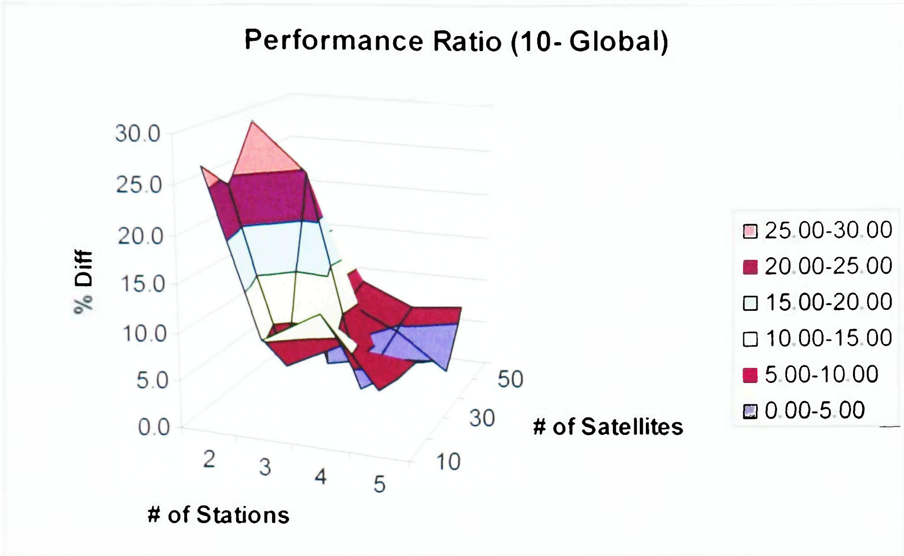

Figure 33 - Percent Difference, Performance, Global Stations, 10/Day

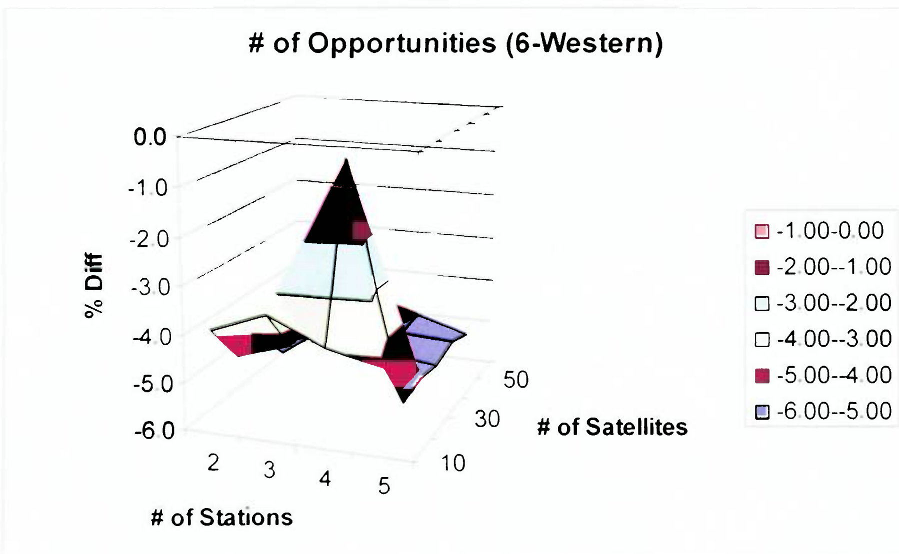

Figure 34 - Percent Difference, Opportunities, Western Hemisphere Stations, 6/Day

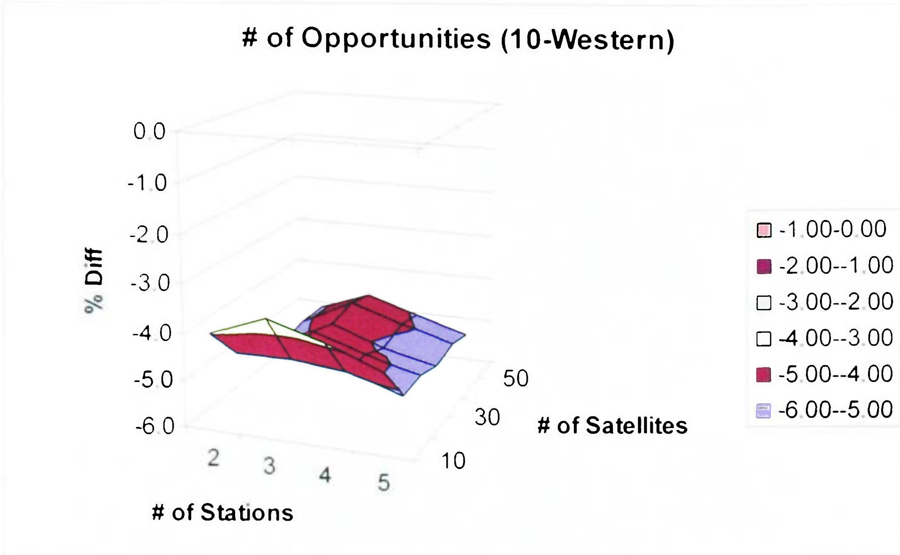

Figure 35 - Percent Difference, Opportunities, Western Hemisphere Stations, 10/Day

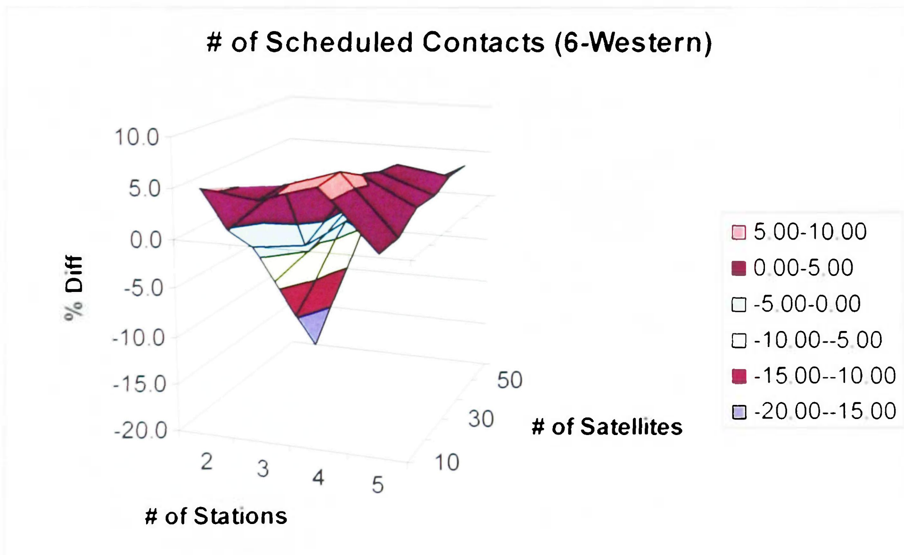

Figure 36 - Percent Difference, Scheduled Contacts, Western Hemisphere Stations, 6/Day

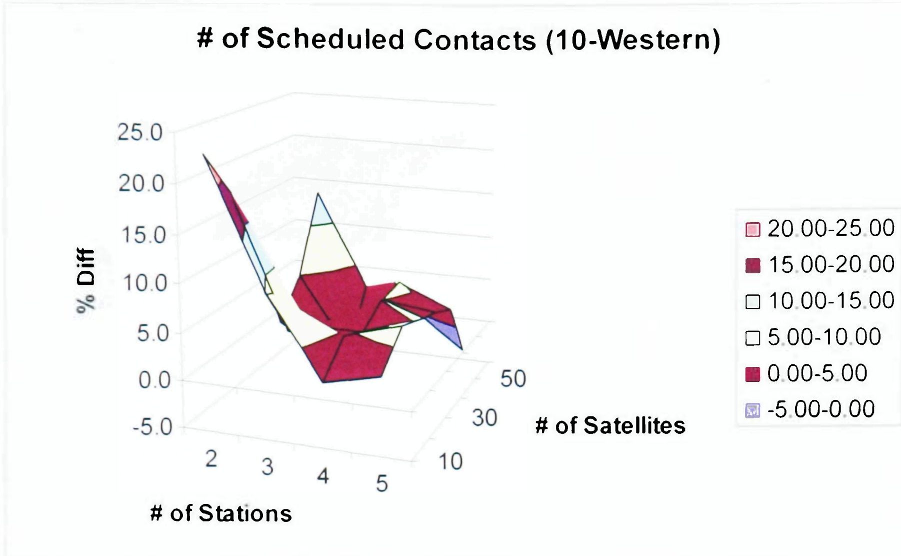

Figure 37 - Percent Difference, Scheduled Contacts, Western Hemisphere Stations, 10/Day

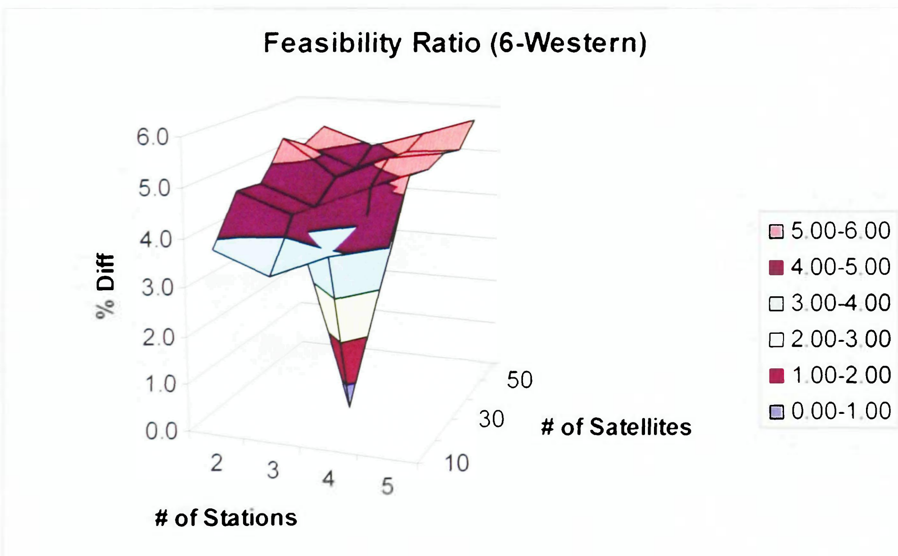

Figure 38 - Percent Difference, Feasibility, Western Hemisphere Stations, 6/Day

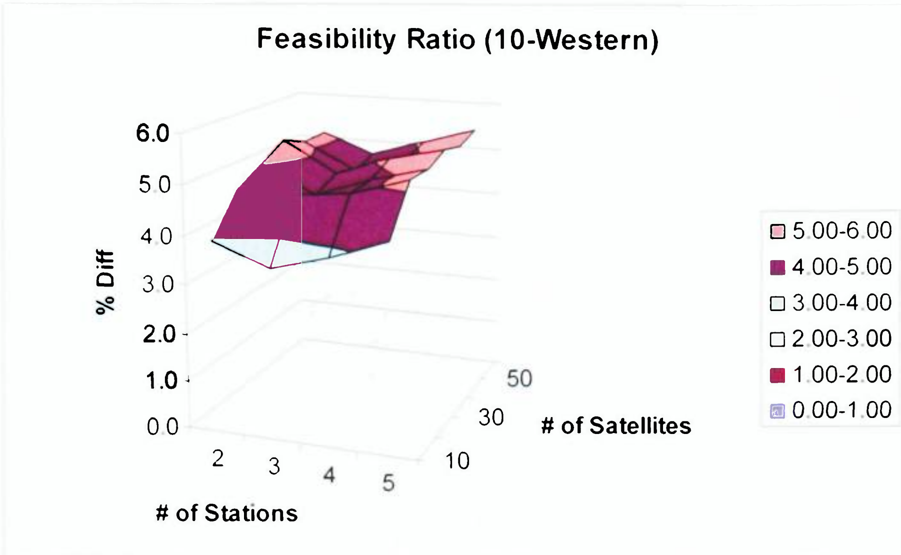

Figure 39 - Percent Difference, Feasibility, Western Hemisphere Stations, 10/Day

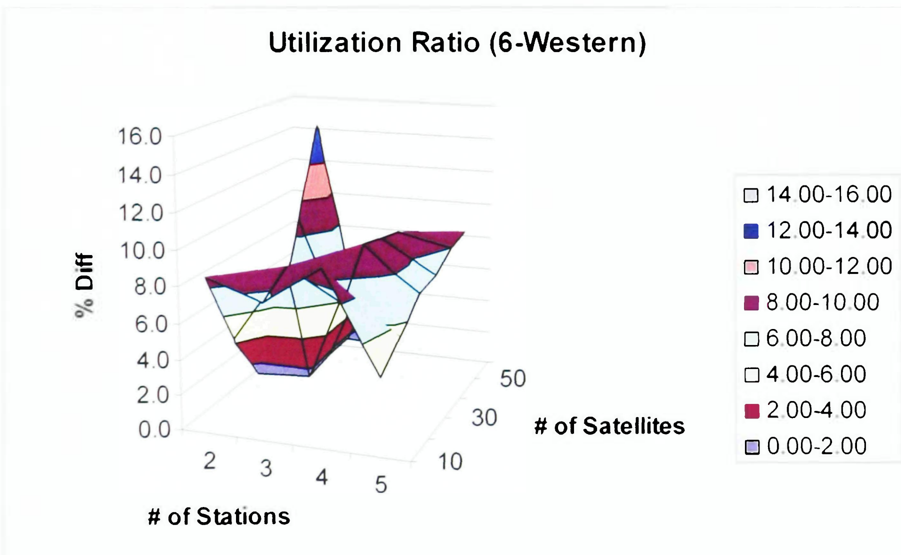

Figure 40 - Percent Difference, Utilization, Western Hemisphere Stations, 6/Day

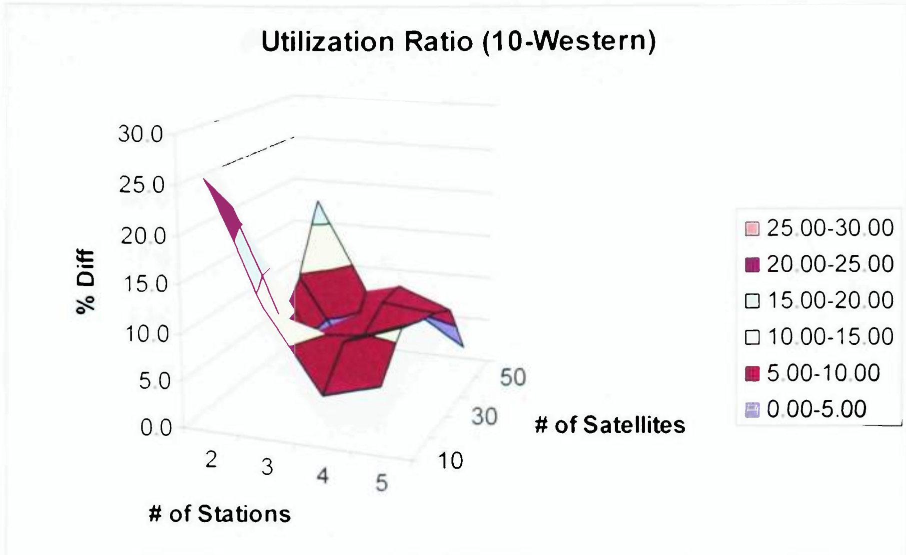

Figure 41 - Percent Difference, Utilization, Western Hemisphere Stations, 10/Day

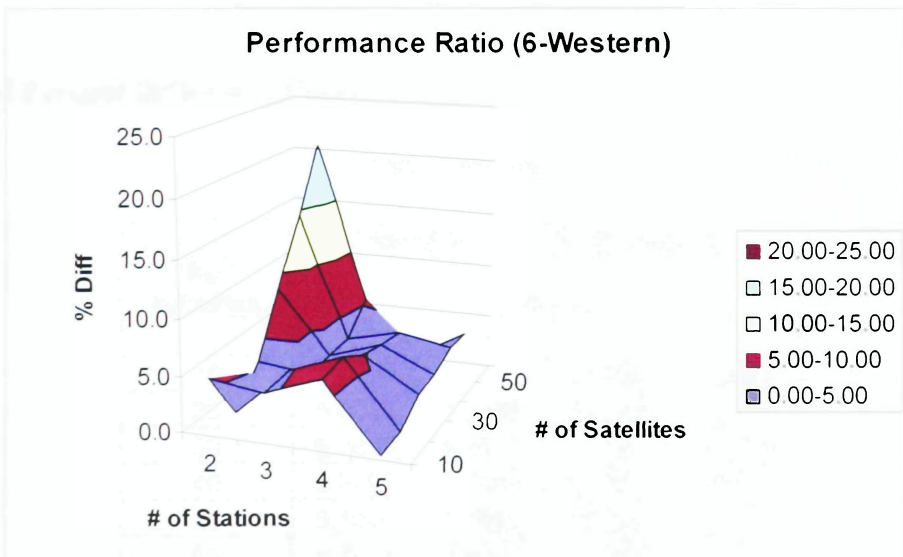

Figure 42 - Percent Difference, Performance, Western Hemisphere Stations, 6/Day

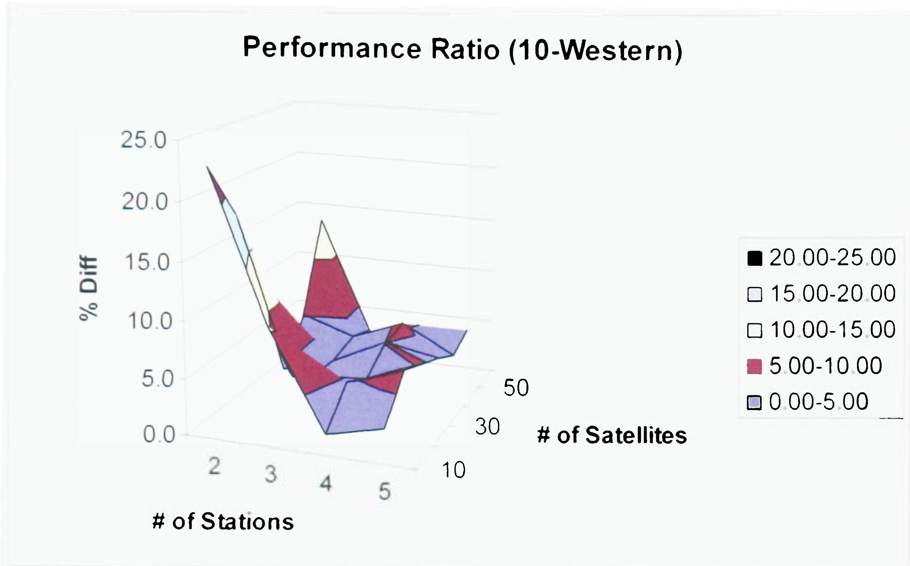

Figure 43 - Percent Difference, Performance, Western Hemisphere Stations, 10/Day

Tables of Percent Difference Values

Global Stations

	Feasibility Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	5.3683	5.2712	5.0068	4.5456
20	5.9025	5.5129	5.6335	5.2770
30	6.0674	5.6631	5.7367	5.3729
40	6.2497	5.9180	6.1091	5.6196
50	6.1646	5.7128	5.8654	5.4977
60	6.3289	5.9262	6.1992	5.7700

Table 46 - % Differenc, Feasibility, Global, 6/day

	Feasibility Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	5.2083	4.6587	5.2818	4.5990
20	6.0089	5.4699	5.6696	5.3242
30	6.0978	5.6486	5.7468	5.3782
40	6.1346	5.9525	6.0775	5.5596
50	6.0569	5.7003	5.8131	5.4210
60	6.1182	5.6202	5.9073	5.5269

Table 47 - % Difference, Feasibility, Global, 10/day

	Utilization Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	7.637632	13.54913	11.98964	9.055008
20	9.441764	10.07097	8.806007	7.712789
30	8.305601	10.6103	8.445007	7.838832
40	0.420697	7.957859	8.282429	7.612228
50	2.124697	3.632511	9.019643	8.056194
60	7.645231	3.66972	6.324835	8.088421

Table 48 -% Difference, Utilization, Global, 6/day

	Utilization Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	30.7840	14.2933	18.1179	10.9202
20	28.4482	10.5359	13.5293	10.6287
30	33.7024	17.4481	7.2792	10.4995
40	25.0686	7.5805	2.6715	8.8484
50	26.7688	12.5098	1.0169	5.7889
60	15.8092	13.2311	0.1213	0.8957

Table 49 - % Difference, Utilization, 10/day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	2.6795	8.9920	7.5830	4.9212
20	4.0967	5.1133	3.6686	2.8429
30	2.7421	5.5480	3.1930	2.8869
40	5.8026	2.5108	2.6794	2.4203
50	8.4201	1.8726	3.6833	3.0013
60	14.4582	9.8133	0.5177	2.7852

Table 50 - % Difference, Performance, Global, 6/day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	27.1791	10.3005	13.7931	6.8234
20	24.1487	5.6423	8.6268	5.8704
30	29.6598	24.6844	1.9507	5.6860
40	20.4719	2.0793	3.2436	3.7809
50	22.3331	7.5224	4.7372	0.6817
60	10.6582	8.3546	5.7787	6.4719

Table 51 - % Difference, Performance, Global, 10/day

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	-5.3560	-5.2623	-5.0132	-4.5456
20	-5.8996	-5.5086	-5.6365	-5.2783
30	-6.0655	-5.6660	-5.7367	-5.3721
40	-6.2497	-5.9179	-6.1083	-5.6209
50	-6.1635	-5.7110	-5.8666	-5.4977
60	-6.0317	-5.6334	-5.9047	-5.4767

Table 52 - % Difference, Opportunities, Global, 6/day

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	-5.2021	-4.6631	-5.2850	-4.5990
20	-6.0117	-5.4656	-5.6666	-5.3242
30	-6.0940	-5.6515	-5.7468	-5.3765
40	-6.1332	-5.9503	-6.0782	-5.5609
50	-6.0546	-5.7021	-5.8138	-5.4209
60	-6.1163	-5.6187	-5.9072	-5.5264

Table 53 - % Difference, Opportunities, Global, 10/day

	# of Scheduled Contacts (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	2.6915	9.0132	7.5830	4.9107
20	4.1090	5.1242	3.6794	2.8322
30	2.7328	5.5407	3.2002	2.8833
40	-5.7983	2.5137	2.6739	2.4257
50	-8.4162	-1.8781	3.6789	3.0014
60	-14.1403	-9.5029	0.7941	3.0571

Table 54 - % Difference, Scheduled Contacts, Global, 6/day

	# of Scheduled Contacts (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	27.1722	10.3139	13.7931	6.8169
20	24.1571	5.6499	8.6235	5.8704
30	29.6598	4.0272	1.9532	5.6882
40	20.4782	2.0738	-3.2436	3.7845
50	22.3303	7.5225	-4.7392	0.6833
60	10.6522	8.3547	-5.7786	-6.4719

Table 55 - % Difference, Scheduled Contacts, Global, 10/day

Western Hemisphere Stations

	Feasibility Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	3.8320	3.3909	3.9069	4.1759
20	4.7429	4.3636	0.4586	5.2895
30	4.6895	4.2508	4.7472	5.2130
40	5.4432	4.6561	5.1684	5.3350
50	5.1215	4.6277	5.0763	5.1818
60	5.3208	4.9550	5.2838	5.6724

Table 56 - % Difference, Feasibility, Western, 6/day

	Feasibility Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	3.9514	3.5154	3.8371	4.2634
20	4.7293	4.7183	4.8342	5.1798
30	5.0305	4.4746	4.8593	5.1776
40	5.3306	4.6807	5.1066	5.3701
50	5.1216	4.5462	4.9105	5.2374
60	5.1328	4.7036	5.0559	5.4104

Table 57 - % Difference, Feasibility, Western, 10/day

	Utilization Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	8.7156	7.6371	9.7909	4.3638
20	4.2849	8.7475	6.9615	5.4839
30	1.2483	1.3889	9.4456	6.9944
40	3.1318	1.6795	9.0960	7.1493
50	8.9645	1.6807	8.6820	7.9690
60	14.2451	0.5048	4.8742	8.2314

Table 58 - % Difference, Utilization, Western, 6/day

	Utilization Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	26.03	13.54	5.47	7.07
20	21.93	8.74	9.04	11.04
30	15.47	7.15	7.74	9.91
40	6.18	1.91	9.65	9.13
50	9.77	1.00	9.82	7.66
60	17.60	6.99	5.14	1.41

Table 59 - % Difference, Utilization, Western, 10/day

	Performance Ratio (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	5.22	4.51	6.27	0.37
20	0.25	4.77	6.53	0.48
30	3.38	2.80	5.15	2.15
40	8.75	2.90	4.40	2.20
50	14.55	2.87	4.05	3.20
60	20.32	5.48	0.15	3.03

Table 60 - % Difference, Performance, Western, 6/day

	Performance Ratio (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	23.1094	10.4959	1.8393	3.1033
20	18.2427	4.4329	4.6379	6.4325
30	11.2156	3.0001	3.2517	5.2441
40	1.1768	2.6774	5.0370	4.2499
50	5.1463	3.4999	5.3904	2.8196
60	13.3705	2.6150	0.3446	3.9272

Table 61 - % Difference, Performance, Western, 10/day

	# of Opportunities (6 Contact/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	-3.8254	-3.3909	-3.9069	-4.1705
20	-4.7497	-4.3662	-0.4553	-5.2923
30	-4.6872	-4.2525	-4.7473	-5.2139
40	-5.4450	-4.6548	-5.1703	-5.3343
50	-5.1230	-4.6255	-5.0763	-5.1812
60	-5.0283	-4.6635	-4.9900	-5.3779

Table 62 - % Difference, Opportunities, Western, 6/day

	# of Opportunities (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	-3.9580	-3.5106	-3.8371	-4.2662
20	-4.7327	-4.7132	-4.8342	-5.1826
30	-5.0258	-4.4763	-4.8568	-5.1767
40	-5.3270	-4.6821	-5.1047	-5.3694
50	-5.1230	-4.5483	-4.9113	-5.2363
60	-5.1316	-4.7045	-5.0566	-5.4114

Table 63 - % Difference, Opportunities, Western, 10/day

	# of Scheduled Contacts (6 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	5.2061	4.5051	6.2457	0.3700
20	-0.2478	4.7655	6.5453	0.4788
30	-3.3717	-2.7936	5.1541	2.1498
40	-8.7558	-2.8944	4.4004	2.1986
50	-14.5355	-2.8690	4.0418	3.2000
60	-19.9943	-5.1918	0.1217	3.2916

Table 64 - % Difference, Scheduled Contacts, Western, 6/day

	# of Scheduled Contacts (10 Contacts/Day)			
# of Satellites	# of Stations			
	2	3	4	5
10	23.1095	10.5027	1.8393	3.0966
20	18.2528	4.4419	4.6380	6.4357
30	11.2112	3.0036	3.2464	5.2485
40	1.1676	-2.6807	5.0347	4.2481
50	5.1383	-3.5030	5.3904	2.8229
60	13.3737	2.6098	0.3447	-3.9255

Table 65 - % Difference, Scheduled Contacts, Western, 10/day